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# THE COMPLETE BALMER SERIES IN THE SODIUM SPECTRUM

By R. W. WOOD

The largest number of lines forming a Balmer series which has been observed in the spectrum of a terrestrial source occurs in the case of hydrogen, thirteen lines having been found by Cornu and by Ames (Kayser's *Spectroscopie*, **2**, 505). In the case of the hydrogen spectrum of the solar chromosphere Evershed (*ibid.*, p. 506) found 29 lines, the last line corresponding to $n=31$ in the formula. In the case of sodium but seven lines have been previously known, belonging to the principal series. As I have shown in a paper recently published (*Phil. Mag.* [6], **16**, 945, December 1908) it is possible to follow the series much farther in the absorption spectrum, and I gave provisional wave-lengths to the 24 lines which had been observed at the time of writing the paper.

In this paper I ventured the opinion that we were dealing with the complete series, the limit depending merely upon the resolving power of the spectroscope and the vapor-density. This conjecture has been verified by some observations which I have recently made with the large quartz spectrograph at the Bureau of Standards, through the courtesy of Director Stratton. Dr. P. G. Nutting of the bureau very kindly placed his laboratory at my disposal for several days and worked with me upon the problem. The sodium was contained in a seamless tube of thin steel, which was exhausted to a pressure of a millimeter or two, and heated to a full red heat in a combustion

furnace. The spectrograph had a focal length of a meter and was furnished with a large Cornu double prism of quartz. The first photograph which we made showed about forty lines, an increase of twelve. We used as a source of light a very powerful cadmium spark obtained from a 10,000-volt transformer with a condenser in the circuit. An exposure of about an hour was sufficient, with a column of sodium vapor 80 cm in length. There were indications, as before, that more lines would come out with a higher resolving power, and we accordingly added two more Cornu prisms, which gave us a dispersion and resolving power comparable with that of a 21-foot grating in the first order. An iron arc comparison spectrum was impressed on each plate, with an exposure of 15 seconds.

It was found that a good deal depended upon getting the sodium vapor at just the right density. The color of the transmitted light was deep violet, but if the vapor became too dense a general absorption appeared in the ultra-violet which spoiled the spectrum. On the best plate I have counted 48 lines, which enables us to test the formula up to the point where $n=50$. This puts solar hydrogen far in the rear with its 29 lines.

Three of the spectra are reproduced on Plate XII. They were taken with different degrees of vapor-density. In the case of the rarer vapor the end of the series does not come into view. It is interesting to note the progressive decrease in the width of the lines with decreasing wave-length. The first five lines of the series are not reproduced, as the scale is too large. With dense vapor they are very broad, and a channeled absorption spectrum was found bordering the first three lines in the ultra-violet, analogous to the one which is found both above and below the D lines. As I have shown in a previous paper published in the *Philosophical Magazine*, it is probable that the mechanism which produces the principal series is identical or connected with that which gives rise to the complicated channeled absorption spectrum, with its thousands of lines, for the absorption of blue-green light by the vapor causes the yellow D lines to appear in the fluorescence spectrum. It is most significant that the other lines of the principal series are bordered by channeled spectra as well as the D lines.

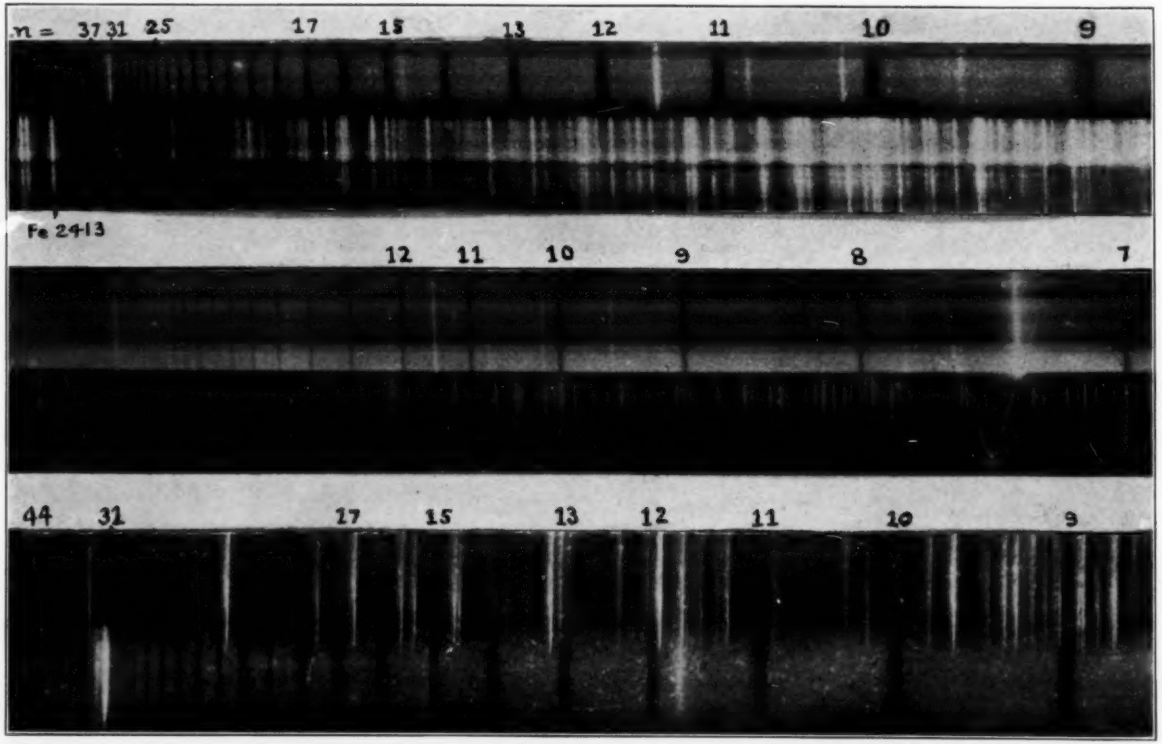
Even on the best plate, on which it is possible to count 48 lines,

PLATE XII

1 Na Fe

2 Na Fe

3 Fe Na




BALMER SERIES IN ABSORPTION SPECTRUM OF SODIUM VAPOR

there are indications that higher resolving powers would bring more into view, for the head of the band is clearly indicated, though it is not resolvable into lines.

The plates were very carefully measured on the large dividing engine of the university, and the results agreed within about 0.02 of an Ångström unit. They are given in the following table.

WAVE-LENGTHS OF LINES IN BALMER SERIES OF SODIUM VAPOR

| $n$ | $\lambda$ | | $n$ | $\lambda$ | $n$ | $\lambda$ |
|---|---|---|---|---|---|---|
| 3 | 5896.16 | Previous observations | 18 | 2433.85 | 35 | 2417.38 |
| | 90.19 | | 19 | 31.43 | 36 | 17.10 |
| 4 | 3303.07 | | 20 | 29.42 | 37 | 16.80 |
| | 02.47 | | 21 | 27.72 | 38 | 16.56 |
| 5 | 2852.91 | | 22 | 26.28 | 39 | 16.33 |
| 6 | 2680.46 | | 23 | 25.00 | 40 | 16.11 |
| 7 | 2594.05 | | 24 | 23.88 | 41 | 15.89 |
| 8 | 43.82 | | 25 | 22.90 | 42 | 15.70 |
| 9 | 12.15 | | 26 | 22.04 | 43 | 15.52 |
| 10 | 2490.70 | | 27 | 21.29 | 44 | 15.37 |
| 11 | 75.60 | | 28 | 20.60 | 45 | 15.21 |
| 12 | 64.53 | | 29 | 20.02 | 46 | 15.06 |
| 13 | 56.02 | | 30 | 19.50 | 47 | 14.94 |
| 14 | 49.46 | | 31 | 19.00 | 48 | 14.78 |
| 15 | 44.24 | | 32 | 18.44 | 49 | 14.64 |
| 16 | 40.06 | | 33 | 18.09 | 50 | 14.50 |
| 17 | 36.70 | | 34 | 17.71 | | |

As will be seen from the table, the last 22 lines, i. e., nearly one-half of the whole series thus far observed, fall in a region of the spectrum not wider than the distance between the D lines!

Kayser gives on page 521 the constants of the Balmer formula for the sodium series, computed from the seven lines known at the time.

These seven lines are well represented by the expression

$$10^8\lambda^{-1}=41496.34-127040\,n^{-2}-843841\,n^{-4}.$$

With the complete series now at our disposal it will be necessary to redetermine the constants.

I have calculated the wave-lengths of the 32d and the 50th line from the formula, finding 2417.5 instead of 2418.44 (observed), and 2415.2 instead of 2414.5; that is, the calculated values are separated from the observed by a distance equal to the distance between four of the lines at this point of the spectrum. As will be seen from

the table, the distance between the last lines is only 0.16 of an Ångström unit, and we are probably within one unit of the theoretical limit of the series.

I intend to make an attempt to extend the series still farther by photographing the spectrum with the 21-foot grating in the second order, though a very long exposure will be necessary, as a screen will have to be used which will cut out the superposed blue-green of the first order. Such a screen is not difficult to prepare, though its use may increase the necessary time of exposure.

One point of great interest noticed with very dense sodium vapor is the general absorption, which begins exactly at the head of the Balmer series and extends from this point down to the end of the ultra-violet. The vapor is much more transparent to the light between the absorption lines than in the region below the head of the series. The head of the series in the absorption spectrum actually shows much *brighter* on this account than the rest of the spectrum below it, exactly the opposite of what we should expect. Evershed's observation of a faint continuous *emission* spectrum below the head of the hydrogen series (chromosphere) is interesting in this connection. This matter in relation to the density of the vapor and the admixture of small traces of other gases will be further investigated.

Johns Hopkins University
December 9, 1908

# ON THE APPARENT DISPERSION OF LIGHT IN SPACE

By PETER LEBEDEW

The discovery, on the one hand, of delays in the time of coincidence of lines, made by Belopolsky[1] in the case of the spectroscopic binary *β Aurigae*, and, on the other hand, the detection of certain photometric peculiarities of variable stars, made almost simultaneously with different apparatus by Nordmann[2] and by Tikhoff,[3] have led these gentlemen to explain the observed phenomena as the result of an appreciable dispersion of light in space. This assumption seemed to me so improbable that I preferred[4] to ascribe the observed phenomena to physical peculiarities of the observed stars, and not to new properties of space; this led to a lively exchange of opinions on the subject.[5]

Since the question of the dispersion of light in space is of significance in principle, it seems to me that a discussion of this matter is not superfluous.

## I. DISPERSION OF LIGHT IN SPACE AND THE ELECTROMAGNETIC THEORY OF LIGHT

If we adopt the standpoint of the electromagnetic theory of light, which has been proved many times experimentally and has maintained itself so brilliantly, we can assign a dispersion only to a medium composed of the dispersionless ether and the selectively absorbing molecules imbedded therein. In such a medium the ***dispersion is inseparably connected with the absorption of light.***[6] Planck[7] has

[1] *Bulletin de l'Acad. de St. Pétersbourg,* **21**, 153, 1905 (in Russian).

[2] *Comptes Rendus,* **146**, 266 and 383, 1908.

[3] *Ibid.,* **146**, 570, 1908; *Mitteilungen der Nikolaisternwarte in Pulkowa,* No. 21, 1908.

[4] *Bulletin de l'Acad. de St. Pétersbourg,* **24**, 93, 1906 (in Russian); *Comptes Rendus,* **146**, 1254; **147**, 515, 1908.

[5] A. Belopolsky, *Bulletin de l'Acad. de St. Pétersbourg,* **24**, 97, 1906 (in Russian); C. Nordmann, *Comptes Rendus,* **147**, 24 and 620, 1908; G. A. Tikhoff, *ibid.,* **147**, 170, 1908; J. Stein, *ibid.,* **147**, 228, 1908.

[6] The connection between dispersion and absorption is not limited to solely electromagnetic processes, occurring for all vibrations, both elastic and acoustic.

[7] *Sitzungsberichte der Berliner Acad.,* **1904**, 748.

computed the distance of relaxation, i. e., the thickness of the stratum in which the amplitude of a plane wave is reduced to its $e^{th}$ part for a medium in which the dispersion corresponds to that of hydrogen at 760 mm and 0° C.

| Line | Index | Distance of Relaxation |
|---|---|---|
| B | 1.00014217 | $1.8\times 10^8$ cm |
| D | 1.00014294 | $1.0\times 10^8$ cm |
| G | 1.00014554 | $2.7\times 10^7$ cm |

The delay of light between the lines D and G, which according to Nordmann and Tikhoff occurs in space, would have to be from 0.000,000,01 to 0.000,000,34; that is, it would be equal to the dispersion in hydrogen at 0° C. and at from 3 mm to 100 mm pressure. For hydrogen at 3 mm pressure, the relaxation-distance for the D line would amount to about $2.5\times 10^9$ cm. But through such a medium no appreciable fraction of the light of the sun or of the stars could in any way reach the earth. If for hydrogen we substitute any other dispersive gas, and if we make the assumption that this gas is uniformly distributed, or not, in the direction of the ray, the final result is thereby not altered: the sun and the stars would remain invisible. The possibility is therefore excluded that demonstrable dispersion in space can be caused by the presence of ponderable matter subject to known physical laws.

There would be left the hypothesis that the ether itself might produce a dispersion without absorption. This assumption, however, is equivalent to asserting that all our electromagnetic theories must be cast aside at once, since the question as to the dispersion of light in pure ether is by no means one which has been overlooked or has not received attention in scientific investigation. The assumption that there is ***absolutely no dispersion*** in pure ether forms the ***basis of all the electromagnetic theories***, and countless experimental results of the most varied sorts (cathode rays, Röntgen rays, and everything in optics) are in entire harmony with this assumption. It is therefore not permissible to propose that such a dispersion exists in space as a provisional astronomical hypothesis to satisfy observations; it could be proposed only after a mature critical examination of its foundation and with a clear idea of its enormous significance.

In what follows I shall attempt to show that positive evidence of a dispersion of light in space, which Nordmann and Tikhoff believe they have found, gives no occasion for a revision of our electromagnetic theories, inasmuch as the ***methods employed are on principle inapplicable*** and the positive result announced is ***illusory***.

## II. THE SPECTROSCOPIC BINARY $\beta$ *Aurigae*

Tikhoff employed the spectroscopic data as to this star obtained by Belopolsky and himself in the following manner. From spectrograms made for two regions, the "blue" ($\lambda$ 442 to $\lambda$ 462), and the "violet" (from $\lambda$ 393 to $\lambda$ 410), he computes the epochs of coincidence of the lines, and he finds that for the "violet" rays the epoch is delayed by 0.015 day with respect to the blue rays; from this Tikhoff concludes that there is a demonstrable amount of dispersion of light in space.

In his computation he makes the tacit hypothesis that all "blue" spectral lines and all the "violet" lines, respectively, coincide simultaneously (i. e., differ by less than 0.005 day and are free from systematic errors). This hypothesis is false, however, and consequently Tikhoff's whole computation is illusory, and his publication already cited here gives only mean values from which the error committed can no longer be proved. In order to show this error clearly, I give in Table I the direct measurements,[1] limiting myself to three "violet" lines ($\lambda_1=393$, *Ca;* $\lambda_2=405$, *Fe;* $\lambda_3=408$ ?), for which the observed data is most extensive in the neighborhood of the second epoch of coincidence, at about 2.00 days. The material is collected for different epochs, $E$, which are reckoned forward in days from the moment of the first coincidence; under $v_1$, $v_2$, $v_3$, the differences in kilometers per second of the measured velocities of the two components according to the measures of Belopolsky ($B$) and Tikhoff ($T$) are given. As Tikhoff states, the velocities ($v$) are well represented by the formula

$$v=222 \text{ km} \sin \frac{360\times E}{3.9599} .$$

[1] I extract these original measurements from the hardly accessible paper by Tikhoff, printed in Russian, "Attempt at an Investigation of the Dispersion in Space on the Basis of Observations of the Spectroscopic Binary $\beta$ *Aurigae*" in the ***Publications of the School of Mines*** in Jekaterinoslaw, 1905, pp. 32, 33.

With this formula I have computed the epochs $C_1$, $C_2$, $C_3$, of the coincidence of the lines from each separate observation, and I have formed the differences $C_1 - C_2$ and $C_2 - C_3$ for each plate.

TABLE I

| E | | $\lambda_1 = 393\ \mu\mu$ $v_1$ | $\lambda_2 = 405$ $v_2$ | $\lambda_3 = 408$ $v_3$ | $C_1$ | $C_2$ | $C_3$ | $C_1 - C_2$ | $C_2 - C_3$ |
|---|---|---|---|---|---|---|---|---|---|
| d | | km | km | km | d | d | d | d | d |
| 1.407 | T | 181.7 | .... | 179.6 | 2.012 | ..... | 2.001 | ..... | ..... |
| | B | 174.5 | 170.9 | 185.0 | 1.977 | 1.877 | 2.191 | +0.100 | −0.314 |
| 1.480 | T | 160.6 | 150.5 | 162.4 | 1.990 | 1.951 | 1.977 | +0.039 | −0.046 |
| | B | 163.6 | 156.7 | 157.2 | 2.003 | 1.973 | 1.996 | +0.030 | −0.023 |
| 1.585 | T | 144.2 | 130.1 | 136.9 | 2.031 | 1.910 | 2.004 | +0.121 | −0.094 |
| | B | 143.6 | 125.1 | 132.2 | 2.029 | 1.962 | 1.987 | +0.067 | −0.025 |
| 2.249 | T | − 92.3 | − 98.4 | ..... | 1.979 | 1.960 | ..... | +0.019 | ..... |
| 2.366 | T | −118.5 | −129.5 | −103.3 | 2.011 | 1.973 | 2.061 | +0.038 | −0.088 |
| | B | −116.2 | −122.7 | −110.0 | 2.019 | 1.993 | 2.039 | +0.026 | −0.046 |
| 2.404 | T | −145.1 | −132.2 | −130.2 | 1.955 | 2.002 | 2.009 | −0.047 | −0.007 |
| | B | −139.1 | −139.9 | −132.0 | 1.977 | 1.974 | 2.003 | −0.003 | −0.029 |
| 2.442 | T | −146.1 | −138.1 | −142.3 | 1.989 | 2.019 | 1.989 | −0.030 | −0.030 |
| 2.583 | T | −173.0 | −171.1 | ..... | 2.020 | 2.028 | ..... | −0.008 | ..... |
| | B | ? | −184.6 | −157.3 | ? | 1.969 | 2.087 | ..... | −0.118 |
| 2.629 | T | −184.3 | −182.9 | ..... | 2.012 | 2.017 | ..... | −0.005 | ..... |
| 2.632 | T | −144.4 | −196.7 | −178.3 | 2.186 | 1.946 | 2.044 | +0.240 | −0.098 |
| | B | −157.3 | −198.1 | −181.5 | 2.136 | 1.935 | 2.029 | +0.201 | −0.093 |

How we may now compute the mean values of $C_1 - C_2$ and $C_2 - C_3$, which observations would appear too discordant and uncertain, is of no consequence for the result. There is unquestionably a systematic difference of the epochs of coincidence $C_1 - C_2$ of about +0.02 day, and for $C_2 - C_3$ of about −0.04 day, which applies to the measurements of each observer on exposures made simultaneously on the same plate.

Hence it follows that it is not permissible to combine the results for separate "violet" lines in a mean value, as Tikhoff has done, and comparing it with a similar mean value for the "blue" lines (which were on different plates made with different instruments and at different times), to thence infer a difference of 0.015 day, inasmuch as this difference can have no real physical meaning. It is sufficiently evident from the data cited that the method proposed by

Tikhoff[1] of using the difference of epochs of coincidence[2] for the investigation of the dispersion of light in space is ***invalid on principle:*** otherwise we should be compelled to assert that there was a "normal" dispersion in the region from $\lambda$ 393 to $\lambda$ 405, and an "anomalous" dispersion in the region from $\lambda$ 405 to $\lambda$ 408.

It is obvious that the observed difference of the epochs of coincidence is to be sought in the individual peculiarities of the star and not in those of space.

### III. THE DISPLACEMENT OF MONOCHROMATIC MINIMA OF VARIABLE STARS

From observations of variable stars with color-filters in approximately monochromatic light, Nordmann by direct photometric measurements, and Tikhoff by estimates of the photographic densities, found in agreement with each other that the epochs of minima in yellow light appreciably anticipate those in violet light.

Thus Nordmann finds for *β Persei* a difference in the time of minima of 16 minutes between $\lambda$ 680 and $\lambda$ 430, and of 9 minutes between $\lambda$ 510 and $\lambda$ 430. From this a difference of time of 11 minutes may be computed for the interval between $\lambda$ 560 and $\lambda$ 430. Tikhoff finds for *RT Persei* a difference in time of 4 minutes for the same spectral interval.

But if we place the results of the determinations of dispersion by the two observers side by side, and note that according to Nordmann the distance of *β Persei* amounts to 60 light-years, while *RT Persei* according to Tikhoff[3] has a distance of 740 light-years, we obtain the ratio:

$$\tfrac{11}{60}:\tfrac{4}{740}>30:1.$$

A method which yields such widely different results for the same constant cannot be anything else than incorrect and therefore ***wholly inapplicable.***

The results obtained for the other stars observed (*λ Tauri, W Ursae majoris*) do not permit of a comparison, since their parallaxes are unknown.

[1] *Memorie Spett. Ital.*, **27**, 107, 1898.

[2] The comparison of the epochs of coincidence would have a physical basis only for lines of the same spectral series of an element. This is overlooked by Tikhoff.

[3] *Comptes Rendus*, **147**, 171, 1908.

The monochromatic light-curves differ very considerably among themselves. Fig. 1 gives the measurements of Tikhoff for *RT Persei* and *W Ursae majoris*.[1] The fact that the monochromatic curves are *not solely* displaced parallel to themselves, is a proof that this behavior is due to the individual optical properties of the star, or to the imperfection of the methods of observation, and not to the dispersion of light. It is therefore physically impossible without discussing the whole behavior of the curves to extract *solely* the moments of minima and regard their differences as a proof that

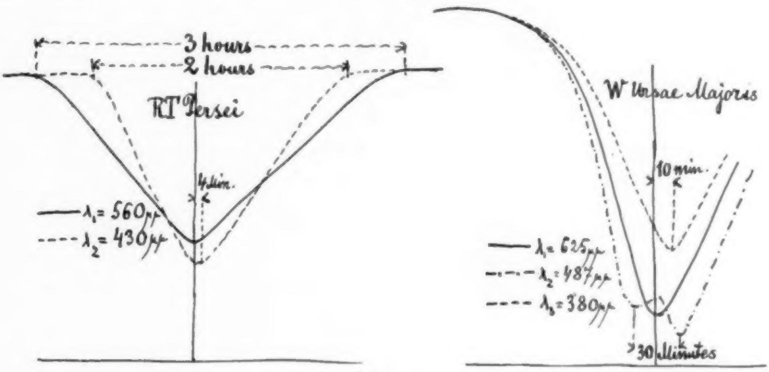



FIG. 1

light is propagated in space with an appreciable dispersion. If we should logically apply this principle further, we should have to regard the assertion as not impossible that the green light of *W Ursae majoris* travels with two different velocities in space.

### IV. ON THE PHYSICS OF VARIABLE STARS

The displacement of the epochs of coincidence of the lines for *β Aurigae*, as well as the displacement of the monochromatic minima for *RT Persei* may be attributed in a natural way to physical processes on the variable stars. In what follows I would call attention

[1] The form and position of the minima as given by Tikhoff (Fig. 1) can only be accepted with reserve: the photographic method is ill adapted to such investigations, as it furnishes the integral effect of a relatively long exposure time (e. g., for *RT Persei*, 10 to 20 minutes), and the fine details of the minima are concealed. For this case direct photometry as employed by Nordmann is decidedly more appropriate, as this furnishes the instantaneous values of the brightness; unfortunately these observations are not yet published.

to two processes which are competent to give to the stars the optical properties already described.

The system of $\beta$ *Aurigae* is according to Vogel[1] composed of two suns which are very close to each other; it is therefore to be assumed that the absorbing masses of gas which surround these suns are under a different pressure on the sides toward each other from that on the sides away from each other. An observer compares

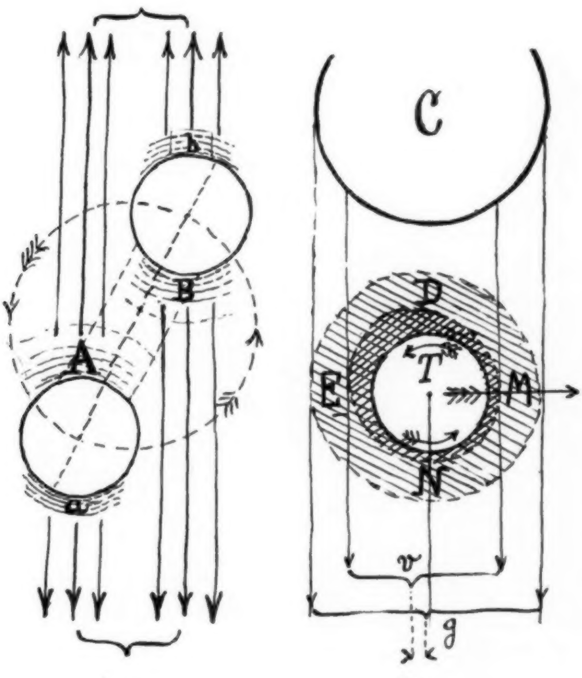



FIG. 2 FIG. 3

the coincidences of lines from two absorbing layers (*a* with *B*, and *A* with *b*, Fig. 2) which are under different degrees of pressure, and are therefore according to Humphreys displaced in the spectrum. The coincidences of lines then will not be observed at the transit of the stars through the line of sight, but at such velocities of the bodies in the line of sight that the Doppler displacement exactly corresponds to the pressure-shift: the epochs of coincidence must

[1] *Sitzungsberichte der Berliner Acad.*, **1904**, 514.

therefore turn out differently for different lines, which is in agreement with the discovery of Belopolsky as to *β Aurigae*.

He satisfactorily shows from the nature of the monochromatic curves of eclipse that the eclipsing body possesses an atmosphere with a selective absorption, for otherwise we should not be able to explain the conspicuous difference of the amplitudes of these curves.

We must assume in the case of *RT Persei* that the violet-absorbing atmosphere which causes an eclipsing during two hours is wrapped closer about the satellite than the yellow-absorbing atmosphere which causes an eclipsing during three hours. If we make the assumption that these atmospheres are not absolutely concentric and that their optical centers of gravity are displaced *by the thirtieth part* of their diameters with respect to each other in the direction of their orbital velocities, then the displacement of the monochromatic minima of four minutes measured by Tikhoff is quantitatively explained.

Such an asymmetry may be explained by the axial revolution of the satellite. The thermal effect of the central body C (Fig. 3) on the daylight side *D* may produce such alterations in the distribution of the atmosphere about the satellite, that an appreciable difference is caused in the boundaries of the violet-absorbing and the yellow-absorbing elements of the atmosphere on the morning side *M* and on the evening side *E* of the satellite.

Such a case occurs on a small scale in our planetary system. If a person on the moon was to observe a solar eclipse both in the infra-red rays at $\lambda$ 3.0 $\mu$, where there is a maximum of absorption of water-vapor, and also in the visible rays, at $\lambda$ 0.5 $\mu$, for which water-vapor is transparent, then he would note a displacement of these monochromatic minima, since the absolute humidity of the earth's atmosphere is greater on the evening side of the earth than on the morning side. This displacement would be measured in fractions of a second for an eclipse by the earth, but in an eclipse by a large satellite moving close to the central body, it might increase to several minutes.

The thermal phenomena are not the only ones which can be here effective: as Nordmann[1] remarks, tidal phenomena in the atmospheres

[1] *Comptes Rendus*, **147**, 24, 1908.

of the central body and of the satellite must similarly produce strong optical asymmetry.

The observed data available are too incomplete to permit us to go further into the constitution of the stars *β Aurigae* or *RT Persei.* The ideas presented above as to the structure of these systems are only intended to serve for assigning to already known physical processes the cause of the phenomena regarded by Nordmann and Tikhoff as the dispersion of light in space. However, since such processes are very complicated for each individual star and are quantitatively entirely unknown to us, it is clear that the methods of Tikhoff and of Nordmann are not competent to separate a dispersion in space, even if existing, from these effects, whence *on principle such a dispersion cannot be demonstrated.*

It follows from what has been said above that now as heretofore, in agreement with all of our experience as to electromagnetic processes, we must regard *space as entirely free from any demonstrable dispersion of light.*

Moscow
November 1908

# SPECTROSCOPIC INVESTIGATIONS OF THE ROTATION OF THE SUN DURING THE YEAR 1908[1]

By WALTER S. ADAMS

The results of a determination of the rotation of the sun based on photographic observations of the displacements of the spectrum lines were published by the writer in November 1907.[2] Two conclusions derived from this investigation were of special importance as bearing on a continuation of the work. The first was that a considerably longer series of observations was necessary in order to furnish reliable evidence in regard to a variation in the rate of the solar rotation. The second was that the rotation rate differed for different lines, making it particularly desirable to include in later determinations lines of elements showing a wide range in the altitude which they attain in the solar atmosphere. The observations made during the present year, accordingly, divide themselves naturally into two parts. The first is a direct continuation of the earlier work on the rotation of the general reversing layer with a view to detecting a possible variation in the rate of rotation during the interval covered. The second is a study of the lines of certain elements which are known from investigations of the chromospheric spectrum to rise to great altitudes in the sun's atmosphere. Preliminary values for one of these lines, the *a* line of hydrogen, have already been published.[3] In the present paper is given a more detailed account of the results of the research.

The photographs obtained during 1906–1907 were made with the Snow telescope and the 18-foot (5.5 m) Littrow spectrograph regularly employed with it for general spectroscopic work. During the autumn of 1907 the tower telescope was completed, and the decidedly superior advantages which it offered for an investigation of this sort led to its use for all observations made after January 1908. A

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 33.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 20; *Astrophysical Journal*, **26** 203, 1907.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 24; *Astrophysical Journal*, **27**, 213, 1908.

detailed description of the tower telescope and the 30-foot (9.1 m) Littrow spectrograph used with it will be found elsewhere.[1] The images of the sun formed by the tower are as a rule considerably superior to those given by the Snow telescope. Not only is the definition of the image better, but there is much greater freedom from changes of focus, and the effect of astigmatism is largely eliminated. The last feature is of especial importance in observations of the rotation, since the effect of astigmatism is to bring upon the slit light from different latitudes on the sun's surface, and consequently to introduce systematic errors into the results. As compared with the Snow telescope the tower has but one serious disadvantage, namely, the color-curve of the objective which is used in place of the concave mirror of the former instrument. With a knowledge of the form of this curve, however, it is merely necessary to make a corresponding allowance in focusing the sun's image upon the slit of the spectrograph, and this has always been done when working in the violet region of the spectrum where the color-curve is comparatively steep. The difference in the size of the visual and the photographic images upon the slit is then readily computed from this difference of focus.

The essential difference between the 30-foot spectrograph and the 18-foot instrument of the Snow telescope is that the former is capable of rotation about a vertical axis. This makes it possible to secure any desired position angle on the sun's surface directly. The spectrograph is provided with a divided circle 30 inches (0.76 m) in diameter upon which the readings of the position angles are made. The spectrograph lens can be focused and the grating rotated by handles which pass up the tube from below, and the frame carrying the plate-holder is arranged so that any required tilt may be given to the plate.

The diagonal prism attachment used to bring the opposite limbs of the sun's image upon the slit is considerably more simple than that employed in the first series of observations, owing to the fact that it is fastened directly to the upper end of the spectrograph, and rotates with it, instead of being itself capable of rotation. A small casting about 8 inches high rests on the main plate of the spectrograph, its

[1] Hale, *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, **27**, 204, 1908.

lower surface coming a short distance above the slit. Four small diagonal prisms are mounted on brass blocks fastened to the under surface of this casting, two immediately above the slit, and the other two below small slots in the casting. The distance between the centers of these slots is equal to the mean diameter of the sun's image. The second pair of prisms are capable of adjustment toward or away from each other, to allow for the variations in the sun's diameter. Each prism is held in a small mounting and provided with adjusting screws. On the top of the casting is an aluminum plate ruled with several concentric circles by means of which the sun's image is centered upon the slit. As soon as the prisms had been accurately adjusted so that the grating was uniformly illuminated, the casting was fastened to the top of the spectrograph, its position being accurately defined by tapered pins. It has been found unnecessary to disturb this adjustment in any way, since it is possible to clean the surfaces of the prisms without removing them from their mountings.

It is evident that in a form of instrument in which the diagonal prisms are fixed the danger of errors arising from unequal illumination of the grating surface is much less than in cases where the prisms themselves are movable. I have been careful to test this adjustment frequently, however, usually immediately before the exposure and again at the end when the spectrograph is rotated 90° from its original position. The ratio of aperture to focal length in the case of the objective of the tower telescope is 1 to 60. In a spectrograph of 30 feet focal length, accordingly, a 6-inch grating would be fully illuminated. The one actually used has a ruled surface only 3.25 inches long, and in the higher orders is inclined at such an angle that a beam less than 3 inches in diameter is sufficient to fill it. The factor of safety evidently is considerable.

The grating used in this spectrograph is the same as that employed in the previous determination, with the 18-foot instrument, and described in the earlier paper. The photographs in the violet region of the spectrum have all been made in the third order of the grating; those of the $\alpha$ line of hydrogen in the second order. Both of these orders are exceptionally bright, and the definition is excellent. The linear scale of the plates in the violet of the third order is 1 mm = 0.56 Ångström.

The settings of the position circle of the spectrograph corresponding to the heliographic latitudes desired are made very readily with the aid of a short table of position angles of the sun's axis. In order to apply these, however, a reference line is necessary, and this has been found by observing the transits of the sun's image across the position circle when the coelostat mirror is rotated. The transits of both limbs are observed and the mean value taken as the transit of the sun's center. The readings on the opposite sides of the circle should, of course, differ by 180° if the image has originally been centered upon the slit. The line joining these points gives the true east and west direction, and the calculation of the desired settings becomes very simple. There is a considerable gain of time in obtaining the direction of the reference line in this way, rather than by allowing the image of the sun to drift across the position circle, and the adjustment of the instrument has been found to be so accurate that no appreciable error is introduced.

In selecting the heliographic latitudes to be observed I have followed the same course as in the earlier series of observations. For the reversing layer an exposure is made at every 15° of latitude between 0° and 90°, and so far as possible for at least one intermediate point in each zone. This gives a total of some twelve to fourteen points from which to determine the velocity curve. In the case of the other lines the observations have been limited to every 15° of latitude. The addition of the exposure at the pole of the sun furnishes a most valuable check upon the condition of the instrument, or any possible disturbance of the slit which may lead to a change in the inclination of the spectrum lines. This is an important advantage which the present series of plates possesses over the previous one made with the 18-foot spectrograph. With the latter instrument it was rarely possible to reach the sun's pole on account of the interference of the small diagonal prisms which brought the light to the slit.

It is unnecessary to describe in detail the method of reduction of the plates, since this differs but little from that given in the earlier paper. The calculations of the latitudes have as before been made with the use of De La Rue's reduction tables, and the quantities to be applied to correct the observed velocities for the departure of the sun's pole from its visible edge, as well as for the earth's motion, have

been taken from Dunér's recent valuable memoir.[1] Most of the reversing layer plates have been measured by Miss Lasby upon the 150 mm Toepfer measuring machine. In the case of the other plates a small Gaertner comparator has also been used. The periodic errors for both instruments are well below the errors of measurement.

The discussion of the results obtained naturally divides itself into two parts. The first deals with the general reversing layer, by which is understood the region in the solar atmosphere at which the absorption takes place that gives rise to the great majority of the narrow lines in the spectrum. The second part deals with those lines upon which special investigations have been made.

## RESULTS FOR THE REVERSING LAYER

In order to facilitate direct comparison with the values obtained during 1906–1907 it is clear that the list of lines upon which the determination is based should be nearly identical with the former list. There are, however, a few lines in the region of the spectrum $\lambda$ 4190 to $\lambda$ 4300, not previously included, to which special interest is attached. These have been added to the list, and at the same time two lines in the former list have been omitted. These gave values close to the mean derived from all the lines, and seemed to possess no particular significance. The revised list of lines is as appears in Table I.

In Table I four lines have been added to the list used in the determinations of 1906–1907. Of these $\lambda$ 4207, like $\lambda$ 4197 and $\lambda$ 4216, belongs to the violet cyanogen fluting. Of the remaining three lines, $\lambda$ 4283 and $\lambda$ 4289 are due to calcium, and are included for purposes of comparison with $\lambda$ 4227, while the line $\lambda$ 4233 is added because of its interesting behavior in the spectrum of the chromosphere. Though assigned in Rowland's table to *Mn* it coincides with a strongly "enhanced" line of *Fe*, and its strength in the chromosphere makes the latter identification the more probable.

The series of plates used in this investigation began in February and continued through October, numbering 33 in all. With the exception of July, at least one plate has been taken during each month,

[1] *Nova Acta Regiae Societatis Scientiarum Upsaliensis*, Ser. 4, Vol. I, No. 6.

TABLE I

| λ | Element | Intensity | Behavior at Limb |
|---|---|---|---|
| 4196.699 | *La* | 2 | Much weakened |
| 4197.257 | *C* | 2 | Slightly weakened |
| 4203.730 | *Cr* | 2 | Strengthened and widened |
| 4207.566 | *C* | 1 N | Weakened |
| 4216.136 | *C* | 1 | Weakened |
| 4220.509 | *Fe* | 3 | Slightly strengthened and widened |
| 4232.887 | *Fe* | 2 | Much strengthened and widened |
| 4233.328 | *Mn* | 4 | Much weakened. This is probably not *Mn*, but "enhanced" line of *Fe* |
| 4257.815 | *Mn* | 2 | Slightly strengthened and widened |
| 4258.477 | *Fe* | 2 | Much strengthened and widened |
| 4265.418 | *Fe* | 2 | Slightly weakened |
| 4266.081 | *Mn* | 2 | Slightly weakened |
| 4268.915 | *Fe* | 2 | Slightly weakened |
| 4276.836 | *—Zr* | 2 | Weakened |
| 4283.169 | *Ca* | 4 | Strengthened and widened |
| 4284.838 | *Ni* | 1 | Slightly weakened |
| 4287.566 | *Ti* | 1 | Slightly strengthened and widened |
| 4288.310 | *Ti, Fe* | 1 | Widened |
| 4289.525 | *Ca* | 4 | Probably slightly strengthened |
| 4290.377 | *Ti* | 2 | Slightly weakened. "Enhanced" line of *Ti* |
| 4290.542 | *Fe* | 1 | Slightly weakened |
| 4291.630 | *Fe* | 2 | Much strengthened |

so that the interval may be said to be covered reasonably well. On account, however, of the great variation in the value of the angle the secant of which enters as a factor in correcting the observed velocities for the tilt of the sun's pole, it has been necessary to limit the observations at the highest latitudes to the time when this quantity was smallest. For this reason the plates giving the mean latitude 79°.2 cover but a short period of time. In the case of some of the intermediate latitudes, as well, the number of observations has been increased toward the end of the series in order to bring the weights of these points into substantial agreement with the others. The observations at 0° and multiples of 15° from that point have been distributed nearly uniformly throughout the series.

In Table II is given a summary of the individual plates in the same form as that used in the previous determination. The velocities are corrected for the earth's motion, and are derived from a mean for all the lines.

The mean values derived from Table II and grouped about thirteen points of latitude will be found in Table V. Before entering upon a discussion of the results, however, it is desirable to

TABLE II

| Number of Plate | Date | Number of Lines | $\phi$ | $v$ | Number of Plate | Date | Number of Lines | $\phi$ | $v$ |
|---|---|---|---|---|---|---|---|---|---|
| | 1908 | | | km | | 1908 | | | km |
| $\omega$ 103 | Feb. 16 | 23 | 0°.2 | 2.072 | $\omega$ $120_2$ | June 2 | 22 | 50°.3 | 1.160 |
| | | | 14.7 | 1.948 | | | | 65.8 | 0.697 |
| | | | 29.6 | 1.639 | | | | 79.3 | 0.254 |
| | | | 44.4 | 1.289 | $\omega$ 128 | June 9 | 22 | 0.5 | 2.049 |
| | | | 59.1 | 0.776 | | | | 14.5 | 1.917 |
| | | | 73.4 | 0.434 | | | | 29.5 | 1.684 |
| $\omega$ $105_1$ | Mar. 10 | 22 | 0.3 | 2.068 | | | | 44.5 | 1.258 |
| | | | 14.6 | 1.982 | | | | 59.5 | 0.829 |
| | | | 29.4 | 1.668 | | | | 74.5 | 0.379 |
| | | | 44.2 | 1.343 | $\omega$ 132 | June 10 | 22 | 4.4 | 2.032 |
| | | | 59.3 | 0.844 | | | | 19.4 | 1.896 |
| | | | 74.3 | 0.408 | | | | 34.4 | 1.596 |
| $\omega$ $105_2$ | Mar. 10 | 22 | 0.4 | 2.073 | | | | 49.4 | 1.169 |
| | | | 15.2 | 1.960 | | | | 64.4 | 0.795 |
| | | | 30.1 | 1.673 | | | | 79.4 | 0.062 |
| | | | 45.1 | 1.305 | $\omega$ 134 | June 11 | 22 | 0.5 | 2.048 |
| $\omega$ 106 | Mar. 10 | 22 | 60.5 | 0.794 | | | | 4.5 | 2.013 |
| | | | 75.0 | 0.416 | | | | 19.5 | 1.759 |
| | | | 0.4 | 2.076 | | | | 34.5 | 1.562 |
| | | | 15.2 | 1.971 | | | | 49.5 | 1.166 |
| | | | 30.1 | 1.659 | | | | 64.5 | 0.707 |
| | | | 45.1 | 1.304 | | | | 79.6 | 0.262 |
| | | | 60.0 | 0.799 | $\omega$ $135_1$ | June 11 | 22 | 0.5 | 2.060 |
| | | | 75.0 | 0.408 | | | | 14.5 | 1.940 |
| $\omega$ 113 | April 8 | 22 | 0.0 | 2.095 | | | | 29.5 | 1.675 |
| | | | 14.9 | 1.967 | | | | 44.5 | 1.245 |
| | | | 29.8 | 1.682 | | | | 59.5 | 0.773 |
| | | | 44.8 | 1.294 | | | | 74.5 | 0.381 |
| | | | 60.7 | 0.800 | | | | 79.5 | 0.263 |
| | | | 75.7 | 0.363 | $\omega$ $135_2$ | June 11 | 22 | 0.5 | 2.038 |
| $\omega$ $117_1$ | May 26 | 22 | 0.6 | 2.056 | | | | 14.5 | 1.943 |
| | | | 14.4 | 1.900 | | | | 29.5 | 1.680 |
| | | | 29.4 | 1.664 | | | | 44.5 | 1.238 |
| | | | 44.6 | 1.301 | | | | 59.5 | 0.772 |
| | | | 60.4 | 0.827 | | | | 74.5 | 0.387 |
| | | | 75.9 | 0.383 | | | | 79.5 | 0.266 |
| $\omega$ $117_2$ | May 26 | 22 | 0.6 | 2.060 | $\omega$ 136 | June 11 | 22 | 0.5 | 2.047 |
| | | | 14.4 | 1.902 | | | | 4.5 | 1.982 |
| | | | 29.4 | 1.664 | | | | 19.5 | 1.848 |
| | | | 44.6 | 1.297 | | | | 34.5 | 1.551 |
| | | | 60.4 | 0.845 | | | | 49.5 | 1.149 |
| | | | 75.9 | 0.388 | | | | 64.5 | 0.657 |
| $\omega$ $120_1$ | June 2 | 22 | 12.5 | 1.965 | | | | 79.5 | 0.254 |
| | | | 2.5 | 2.052 | $\omega$ 146 | Aug. 5 | 22 | 0.5 | 2.034 |
| | | | 17.5 | 1.866 | | | | 14.6 | 1.934 |
| | | | 32.8 | 1.654 | | | | 29.6 | 1.658 |
| | | | 48.3 | 1.172 | | | | 44.9 | 1.266 |
| | | | 63.8 | 0.713 | | | | 60.1 | 0.755 |
| | | | 77.3 | 0.335 | | | | 74.9 | 0.403 |
| $\omega$ $120_2$ | June 2 | 22 | 10.5 | 2.015 | $\omega$ 147 | Aug. 5 | 22 | 0.3 | 2.054 |
| | | | 4.5 | 2.039 | | | | 14.6 | 1.959 |
| | | | 19.5 | 1.842 | | | | 29.6 | 1.687 |
| | | | 34.8 | 1.609 | | | | 44.9 | 1.269 |

TABLE II—*Continued*

| Number of Plate | Date | Number of Lines | $\phi$ | $v$ | Number of Plate | Date | Number of Lines | $\phi$ | $v$ |
|---|---|---|---|---|---|---|---|---|---|
| | 1908 | | | km | | 1908 | | | km |
| $\omega$ 147 | Aug. 5 | 22 | 60°.1 | 0.782 | $\omega$ 165 | Aug. 27 | 22 | 49°.4 | 1.141 |
| | | | 74.9 | 0.403 | | | | 64.9 | 0.694 |
| $\omega$ 148 | Aug. 5 | 22 | 0.3 | 2.056 | $\omega$ 166 | Aug. 27 | 22 | 4.2 | 2.048 |
| | | | 14.6 | 1.959 | | | | 11.6 | 1.985 |
| | | | 29.6 | 1.688 | | | | 19.1 | 1.875 |
| | | | 44.9 | 1.265 | | | | 34.1 | 1.560 |
| | | | 60.1 | 0.805 | | | | 49.4 | 1.149 |
| | | | 74.9 | 0.405 | | | | 64.9 | 0.697 |
| $\omega$ 151 | Aug. 6 | 22 | 0.1 | 2.066 | $\omega$ 179 | Sept. 30 | 22 | 60.1 | 0.863 |
| | | | 0.1 | 2.059 | | | | 60.1 | 0.865 |
| | | | 14.6 | 1.952 | $\omega$ 180 | Sept. 30 | 22 | 60.1 | 0.803 |
| | | | 44.6 | 1.270 | | | | 60.1 | 0.805 |
| | | | 59.8 | 0.812 | $\omega$ 182 | Oct. 9 | 22 | 11.0 | 1.990 |
| | | | 74.9 | 0.416 | | | | 11.0 | 1.991 |
| $\omega$ 161 | Aug. 26 | 22 | 4.1 | 2.044 | | | | 19.0 | 1.843 |
| | | | 10.8 | 1.989 | | | | 33.9 | 1.483 |
| | | | 19.2 | 1.886 | $\omega$ 183 | Oct. 9 | 22 | 33.9 | 1.548 |
| | | | 34.2 | 1.547 | | | | 19.0 | 1.864 |
| | | | 49.4 | 1.081 | | | | 11.0 | 1.992 |
| | | | 65.0 | 0.631 | | | | 11.0 | 1.998 |
| $\omega$ 162 | Aug. 26 | 22 | 4.1 | 2.048 | | | | 19.0 | 1.858 |
| | | | 10.8 | 1.988 | | | | 33.9 | 1.549 |
| | | | 19.2 | 1.880 | | | | 33.9 | 1.545 |
| | | | 34.2 | 1.504 | | | | 19.0 | 1.859 |
| | | | 49.4 | 1.146 | $\omega$ 184 | Oct. 22 | 22 | 4.0 | 2.058 |
| | | | 65.0 | 0.695 | | | | 65.5 | 0.671 |
| $\omega$ 163 | Aug. 26 | 22 | 4.1 | 2.040 | | | | 60.3 | 0.822 |
| | | | 10.8 | 2.002 | | | | 50.0 | 1.115 |
| | | | 19.2 | 1.890 | | | | 50.0 | 1.129 |
| | | | 34.2 | 1.547 | | | | 60.3 | 0.827 |
| | | | 49.4 | 1.151 | | | | 65.5 | 0.667 |
| | | | 65.0 | 0.691 | $\omega$ 185 | Oct. 22 | 22 | 4.0 | 2.059 |
| $\omega$ 164 | Aug. 26 | 22 | 4.1 | 2.031 | | | | 65.5 | 0.649 |
| | | | 10.8 | 1.994 | | | | 50.0 | 1.104 |
| | | | 19.2 | 1.872 | | | | 50.0 | 1.105 |
| | | | 34.2 | 1.557 | | | | 65.5 | 0.656 |
| | | 20 | 49.4 | 1.147 | $\omega$ 186 | Oct. 22 | 22 | 4.0 | 2.056 |
| | | 20 | 65.0 | 0.683 | | | | 65.5 | 0.654 |
| $\omega$ 165 | Aug. 27 | 22 | 4.2 | 2.033 | | | | 50.0 | 1.101 |
| | | | 11.6 | 1.992 | | | | 65.5 | 0.633 |
| | | | 19.1 | 1.880 | | | | 4.0 | 2.062 |
| | | | 34.1 | 1.556 | | | | | |

give the values for the individual lines, using the quantities obtained from all of the plates belonging to the same mean latitude. These are given in Table III. In accordance with the usual notation $\xi$ denotes daily angular motion, and here, of course, corresponds to the sidereal period of rotation.

TABLE III

| $\phi$ | $\lambda$ | No. of Plates | $v$ km | $\xi$ |
|---|---|---|---|---|
| 0°.34......... | 4196.699 | 21 | 2.034 | 14°.44 |
| | 4197.257 | 21 | 2.046 | 14.52 |
| | 4203.730 | 21 | 2.061 | 14.65 |
| | 4207.566 | 21 | 2.051 | 14.56 |
| | 4216.136 | 21 | 2.042 | 14.50 |
| | 4220.509 | 21 | 2.058 | 14.62 |
| | 4232.887 | 21 | 2.066 | 14.68 |
| | 4233.328 | 21 | 2.054 | 14.59 |
| | 4257.815 | 21 | 2.076 | 14.75 |
| | 4258.477 | 21 | 2.069 | 14.71 |
| | 4265.418 | 21 | 2.060 | 14.63 |
| | 4266.081 | 21 | 2.074 | 14.74 |
| | 4268.915 | 21 | 2.072 | 14.72 |
| | 4276.836 | 21 | 2.066 | 14.67 |
| | 4283.169 | 21 | 2.070 | 14.70 |
| | 4284.838 | 21 | 2.069 | 14.69 |
| | 4287.566 | 21 | 2.065 | 14.66 |
| | 4288.310 | 21 | 2.066 | 14.67 |
| | 4289.525 | 21 | 2.070 | 14.70 |
| | 4290.377 | 21 | 2.061 | 14.64 |
| | 4290.542 | 21 | 2.070 | 14.70 |
| | 4291.630 | 21 | 2.071 | 14.71 |
| 4.08......... | 4196.699 | 15 | 2.023 | 14.40 |
| | 4197.257 | 15 | 2.026 | 14.42 |
| | 4203.730 | 15 | 2.034 | 14.48 |
| | 4207.566 | 15 | 2.034 | 14.48 |
| | 4216.136 | 15 | 2.028 | 14.43 |
| | 4220.509 | 15 | 2.047 | 14.57 |
| | 4232.887 | 15 | 2.042 | 14.54 |
| | 4233.328 | 15 | 2.049 | 14.58 |
| | 4257.815 | 15 | 2.052 | 14.60 |
| | 4258.477 | 15 | 2.044 | 14.55 |
| | 4265.418 | 15 | 2.041 | 14.54 |
| | 4266.081 | 15 | 2.048 | 14.57 |
| | 4268.915 | 15 | 2.042 | 14.53 |
| | 4276.836 | 15 | 2.039 | 14.52 |
| | 4283.169 | 15 | 2.037 | 14.50 |
| | 4284.838 | 15 | 2.035 | 14.48 |
| | 4287.566 | 15 | 2.032 | 14.46 |
| | 4288.310 | 15 | 2.033 | 14.47 |
| | 4289.525 | 15 | 2.037 | 14.49 |
| | 4290.377 | 15 | 2.032 | 14.46 |
| | 4290.542 | 15 | 2.040 | 14.52 |
| | 4291.630 | 15 | 2.049 | 14.58 |
| 11.16........ | 4196.699 | 12 | 1.986 | 14.37 |
| | 4197.257 | 12 | 1.987 | 14.37 |
| | 4203.730 | 12 | 2.005 | 14.48 |
| | 4207.566 | 12 | 1.997 | 14.44 |
| | 4216.136 | 12 | 1.991 | 14.41 |
| | 4220.509 | 12 | 2.006 | 14.49 |
| | 4232.887 | 12 | 2.001 | 13.47 |
| | 4233.328 | 12 | 2.003 | 14.48 |
| | 4257.815 | 12 | 2.006 | 14.49 |
| | 4258.477 | 12 | 2.006 | 14.49 |

TABLE III—*Continued*

| $\phi$ | $\lambda$ | No. of Plates | $v$ km | $\xi$ |
|---|---|---|---|---|
| 11°16 ........ | 4265.418 | 12 | 2.004 | 14°48 |
| | 4266.081 | 12 | 2.004 | 14.48 |
| | 4268.915 | 12 | 2.001 | 14.47 |
| | 4276.836 | 12 | 2.010 | 14.55 |
| | 4283.169 | 12 | 1.999 | 14.45 |
| | 4284.838 | 12 | 1.995 | 14.43 |
| | 4287.566 | 12 | 1.997 | 14.43 |
| | 4288.310 | 12 | 1.998 | 14.44 |
| | 4289.525 | 12 | 1.999 | 14.45 |
| | 4290.377 | 12 | 1.976 | 14.30 |
| | 4290.542 | 12 | 2.001 | 14.43 |
| | 4291.630 | 12 | 2.000 | 14.44 |
| 14.86 ........ | 4196.699 | 18 | 1.928 | 14.16 |
| | 4197.257 | 18 | 1.929 | 14.17 |
| | 4203.730 | 18 | 1.946 | 14.29 |
| | 4207.566 | 18 | 1.940 | 14.25 |
| | 4216.136 | 18 | 1.944 | 14.29 |
| | 4220.509 | 18 | 1.946 | 14.29 |
| | 4232.887 | 18 | 1.951 | 14.32 |
| | 4233.328 | 18 | 1.948 | 14.30 |
| | 4257.815 | 18 | 1.957 | 14.36 |
| | 4258.477 | 18 | 1.954 | 14.35 |
| | 4265.418 | 18 | 1.948 | 14.30 |
| | 4266.081 | 18 | 1.949 | 14.31 |
| | 4268.915 | 18 | 1.952 | 14.33 |
| | 4276.836 | 18 | 1.951 | 14.33 |
| | 4283.169 | 18 | 1.951 | 14.33 |
| | 4284.838 | 18 | 1.942 | 14.26 |
| | 4287.566 | 18 | 1.946 | 14.29 |
| | 4288.310 | 18 | 1.954 | 14.35 |
| | 4289.525 | 18 | 1.952 | 14.33 |
| | 4290.377 | 18 | 1.935 | 14.21 |
| | 4290.542 | 18 | 1.956 | 14.36 |
| | 4291.630 | 18 | 1.950 | 14.31 |
| 19.21 ........ | 4196.699 | 14 | 1.862 | 14.00 |
| | 4197.257 | 14 | 1.862 | 14.00 |
| | 4203.730 | 14 | 1.874 | 14.09 |
| | 4207.566 | 14 | 1.878 | 14.12 |
| | 4216.136 | 14 | 1.865 | 14.02 |
| | 4220.509 | 14 | 1.878 | 14.12 |
| | 4232.887 | 14 | 1.878 | 14.12 |
| | 4233.328 | 14 | 1.881 | 14.14 |
| | 4257.815 | 14 | 1.879 | 14.13 |
| | 4258.477 | 14 | 1.875 | 14.09 |
| | 4265.418 | 14 | 1.865 | 14.02 |
| | 4266.081 | 14 | 1.876 | 14.10 |
| | 4268.915 | 14 | 1.872 | 14.06 |
| | 4276.836 | 14 | 1.866 | 14.02 |
| | 4283.169 | 14 | 1.863 | 14.00 |
| | 4284.838 | 14 | 1.864 | 14.01 |
| | 4287.566 | 14 | 1.865 | 14.02 |
| | 4288.310 | 14 | 1.869 | 14.04 |
| | 4289.525 | 14 | 1.866 | 14.03 |
| | 4290.377 | 14 | 1.862 | 14.00 |

TABLE III—*Continued*

| $\phi$ | $\lambda$ | No. of Plates | $v$ km | $\xi$ |
|---|---|---|---|---|
| 19°21 . . . . . . . . | 4290.542 | 14 | 1.872 | 14°08 |
| | 4291.630 | 14 | 1.875 | 14.09 |
| 29.66 . . . . . . . . | 4196.699 | 16 | 1.652 | 13.50 |
| | 4197.257 | 16 | 1.655 | 13.52 |
| | 4203.730 | 16 | 1.678 | 13.70 |
| | 4207.566 | 16 | 1.666 | 13.61 |
| | 4216.136 | 16 | 1.647 | 13.46 |
| | 4220.509 | 16 | 1.671 | 13.65 |
| | 4232.887 | 16 | 1.670 | 13.64 |
| | 4233.328 | 16 | 1.663 | 13.58 |
| | 4257.815 | 16 | 1.676 | 13.69 |
| | 4258.477 | 16 | 1.677 | 13.69 |
| | 4265.418 | 16 | 1.666 | 13.61 |
| | 4266.081 | 16 | 1.674 | 13.66 |
| | 4268.915 | 16 | 1.679 | 13.71 |
| | 4276.836 | 16 | 1.673 | 13.66 |
| | 4283.169 | 16 | 1.672 | 13.65 |
| | 4284.838 | 16 | 1.657 | 13.53 |
| | 4287.566 | 16 | 1.672 | 13.65 |
| | 4288.310 | 16 | 1.676 | 13.69 |
| | 4289.525 | 16 | 1.676 | 13.69 |
| | 4290.377 | 16 | 1.666 | 13.61 |
| | 4290.542 | 16 | 1.683 | 13.75 |
| | 4291.630 | 16 | 1.685 | 13.77 |
| 34.11 . . . . . . . . | 4196.699 | 15 | 1.551 | 13.30 |
| | 4197.257 | 15 | 1.549 | 13.28 |
| | 4203.730 | 15 | 1.566 | 13.42 |
| | 4207.566 | 15 | 1.565 | 13.41 |
| | 4216.136 | 15 | 1.559 | 13.31 |
| | 4220.509 | 15 | 1.572 | 13.48 |
| | 4232.887 | 15 | 1.564 | 13.40 |
| | 4233.328 | 15 | 1.565 | 13.41 |
| | 4257.815 | 15 | 1.572 | 13.48 |
| | 4258.477 | 15 | 1.565 | 13.41 |
| | 4265.418 | 15 | 1.560 | 13.38 |
| | 4266.081 | 15 | 1.565 | 13.41 |
| | 4268.915 | 15 | 1.562 | 13.39 |
| | 4276.836 | 15 | 1.558 | 13.33 |
| | 4283.169 | 15 | 1.556 | 13.31 |
| | 4284.838 | 15 | 1.560 | 13.38 |
| | 4287.566 | 15 | 1.558 | 13.36 |
| | 4288.310 | 15 | 1.563 | 13.40 |
| | 4289.525 | 15 | 1.561 | 13.37 |
| | 4290.377 | 15 | 1.560 | 13.36 |
| | 4290.542 | 15 | 1.568 | 13.41 |
| | 4291.630 | 15 | 1.572 | 13.48 |
| 44.69 . . . . . . . . | 4196.699 | 17 | 1.276 | 12.74 |
| | 4197.257 | 17 | 1.280 | 12.78 |
| | 4203.730 | 17 | 1.289 | 12.87 |
| | 4207.566 | 17 | 1.284 | 12.82 |
| | 4216.136 | 17 | 1.273 | 12.71 |
| | 4220.509 | 17 | 1.290 | 12.88 |
| | 4232.887 | 17 | 1.282 | 12.80 |
| | 4233.328 | 17 | 1.288 | 12.86 |

TABLE III—*Continued*

| $\phi$ | $\lambda$ | No. of Plates | $v$ km | $\xi$ |
|---|---|---|---|---|
| 44°69........ | 4257.815 | 17 | 1.300 | 12°98 |
| | 4258.477 | 17 | 1.295 | 12.93 |
| | 4265.418 | 17 | 1.284 | 12.82 |
| | 4266.081 | 17 | 1.298 | 12.96 |
| | 4268.915 | 17 | 1.293 | 12.91 |
| | 4276.836 | 17 | 1.285 | 12.83 |
| | 4283.169 | 17 | 1.288 | 12.86 |
| | 4284.838 | 17 | 1.283 | 12.81 |
| | 4287.566 | 17 | 1.292 | 12.90 |
| | 4288.310 | 17 | 1.286 | 12.84 |
| | 4289.525 | 17 | 1.298 | 12.96 |
| | 4290.377 | 17 | 1.278 | 12.76 |
| | 4290.542 | 17 | 1.295 | 12.93 |
| | 4291.630 | 17 | 1.295 | 12.93 |
| 49.56........ | 4196.699 | 16 | 1.132 | 12.39 |
| | 4197.257 | 16 | 1.131 | 12.38 |
| | 4203.730 | 16 | 1.145 | 12.54 |
| | 4207.566 | 16 | 1.146 | 12.54 |
| | 4216.136 | 16 | 1.136 | 12.44 |
| | 4220.509 | 16 | 1.153 | 12.62 |
| | 4232.887 | 16 | 1.148 | 12.56 |
| | 4233.328 | 16 | 1.148 | 12.56 |
| | 4257.815 | 15 | 1.143 | 12.50 |
| | 4258.477 | 15 | 1.147 | 12.55 |
| | 4265.418 | 16 | 1.142 | 12.50 |
| | 4266.081 | 16 | 1.148 | 12.56 |
| | 4268.915 | 16 | 1.146 | 12.53 |
| | 4276.836 | 16 | 1.145 | 12.52 |
| | 4283.169 | 16 | 1.145 | 12.53 |
| | 4284.838 | 16 | 1.146 | 12.54 |
| | 4287.566 | 16 | 1.143 | 12.51 |
| | 4288.310 | 16 | 1.149 | 12.57 |
| | 4289.525 | 16 | 1.152 | 12.61 |
| | 4290.377 | 16 | 1.140 | 12.48 |
| | 4290.542 | 16 | 1.152 | 12.60 |
| | 4291.630 | 16 | 1.154 | 12.63 |
| 60.00........ | 4196.699 | 22 | 0.796 | 11.31 |
| | 4197.257 | 22 | 0.798 | 11.33 |
| | 4203.730 | 22 | 0.811 | 11.57 |
| | 4207.566 | 22 | 0.808 | 11.54 |
| | 4216.136 | 22 | 0.802 | 11.40 |
| | 4220.509 | 22 | 0.813 | 11.52 |
| | 4232.887 | 22 | 0.807 | 11.45 |
| | 4233.328 | 22 | 0.811 | 11.52 |
| | 4257.815 | 22 | 0.821 | 11.66 |
| | 4258.477 | 22 | 0.811 | 11.52 |
| | 4265.418 | 22 | 0.812 | 11.53 |
| | 4266.081 | 22 | 0.811 | 11.52 |
| | 4268.915 | 22 | 0.818 | 11.62 |
| | 4276.836 | 22 | 0.817 | 11.61 |
| | 4283.169 | 22 | 0.815 | 11.60 |
| | 4284.838 | 22 | 0.812 | 11.52 |
| | 4287.566 | 22 | 0.815 | 11.56 |
| | 4288.310 | 22 | 0.814 | 11.55 |

TABLE III—*Continued*

| φ | λ | No. of Plates | v km | ξ |
|---|---|---|---|---|
| 60°00........ | 4289.525 | 22 | 0.812 | 11°53 |
| | 4290.377 | 22 | 0.805 | 11.44 |
| | 4290.542 | 22 | 0.816 | 11.59 |
| | 4291.630 | 22 | 0.817 | 11.60 |
| 65.04........ | 4196.699 | 17 | 0.666 | 11.20 |
| | 4197.257 | 17 | 0.669 | 11.25 |
| | 4203.730 | 17 | 0.678 | 11.40 |
| | 4207.566 | 17 | 0.674 | 11.33 |
| | 4216.136 | 17 | 0.672 | 11.27 |
| | 4220.509 | 17 | 0.684 | 11.51 |
| | 4232.887 | 17 | 0.680 | 11.44 |
| | 4233.328 | 17 | 0.683 | 11.49 |
| | 4257.815 | 16 | 0.683 | 11.49 |
| | 4258.477 | 16 | 0.680 | 11.44 |
| | 4265.418 | 17 | 0.680 | 11.44 |
| | 4266.081 | 17 | 0.685 | 11.52 |
| | 4268.915 | 17 | 0.689 | 11.59 |
| | 4276.836 | 17 | 0.683 | 11.49 |
| | 4283.169 | 17 | 0.680 | 11.44 |
| | 4284.838 | 17 | 0.683 | 11.49 |
| | 4287.566 | 17 | 0.680 | 11.44 |
| | 4288.310 | 17 | 0.682 | 11.48 |
| | 4289.525 | 17 | 0.688 | 11.57 |
| | 4290.377 | 17 | 0.678 | 11.40 |
| | 4290.542 | 17 | 0.684 | 11.50 |
| | 4291.630 | 17 | 0.686 | 11.53 |
| 74.90........ | 4196.699 | 17 | 0.382 | 10.41 |
| | 4197.257 | 17 | 0.386 | 10.53 |
| | 4203.730 | 17 | 0.395 | 10.76 |
| | 4207.566 | 17 | 0.389 | 10.60 |
| | 4216.136 | 17 | 0.388 | 10.57 |
| | 4220.509 | 17 | 0.401 | 10.93 |
| | 4232.887 | 17 | 0.399 | 10.88 |
| | 4233.328 | 17 | 0.388 | 10.57 |
| | 4257.815 | 17 | 0.407 | 11.09 |
| | 4258.477 | 17 | 0.401 | 10.93 |
| | 4265.418 | 17 | 0.399 | 10.88 |
| | 4266.081 | 17 | 0.405 | 11.04 |
| | 4268.915 | 17 | 0.407 | 11.09 |
| | 4276.836 | 17 | 0.403 | 10.99 |
| | 4283.169 | 17 | 0.403 | 10.99 |
| | 4284.838 | 17 | 0.409 | 11.14 |
| | 4287.566 | 17 | 0.401 | 10.93 |
| | 4288.310 | 17 | 0.403 | 10.98 |
| | 4289.525 | 17 | 0.399 | 10.88 |
| | 4290.377 | 17 | 0.394 | 10.73 |
| | 4290.542 | 17 | 0.405 | 11.04 |
| | 4291.630 | 17 | 0.402 | 10.99 |
| 79.16........ | 4196.699 | 7 | 0.268 | 10.10 |
| | 4197.257 | 7 | 0.270 | 10.18 |
| | 4203.730 | 7 | 0.271 | 10.20 |
| | 4207.566 | 7 | 0.272 | 10.25 |
| | 4216.136 | 7 | 0.263 | 9.90 |
| | 4220.509 | 7 | 0.268 | 10.10 |

TABLE III—*Continued*

| $\phi$ | $\lambda$ | No. of Plates | $v$ km | $\xi$ |
|---|---|---|---|---|
| 79°16 ........ | 4232.887 | 7 | 0.263 | 9°98 |
| | 4233.328 | 7 | 0.271 | 10.19 |
| | 4257.815 | 7 | 0.282 | 10.62 |
| | 4258.477 | 7 | 0.269 | 10.12 |
| | 4265.418 | 7 | 0.279 | 10.50 |
| | 4266.081 | 7 | 0.282 | 10.60 |
| | 4268.915 | 7 | 0.278 | 10.47 |
| | 4276.838 | 7 | 0.277 | 10.42 |
| | 4283.169 | 7 | 0.275 | 10.38 |
| | 4284.838 | 7 | 0.272 | 10.25 |
| | 4287.566 | 7 | 0.275 | 10.37 |
| | 4288.210 | 7 | 0.282 | 10.63 |
| | 4289.525 | 7 | 0.281 | 10.56 |
| | 4290.377 | 7 | 0.272 | 10.25 |
| | 4290.542 | 7 | 0.272 | 10.24 |
| | 4291.630 | 7 | 0.279 | 10.49 |

The behavior of individual lines will be best shown if we form the difference in the value of $\xi$ between each line and the mean of all the lines.

These differences are found in the short summary which follows:

TABLE IV

| $\lambda$ | 0°3 | 4°1 | 11°2 | 14°9 | 19°2 | 29°7 | 34°1 | 44°7 | 49°6 | 60°0 | 65°0 | 74°9 | 79°2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4196.699 | −0°2 | −0°1 | −0°1 | −0°1 | −0°1 | −0°1 | −0°1 | −0°1 | −0°1 | −0°2 | −0°2 | −0°4 | −0°2 |
| 4197.257 | −0.1 | −0.1 | −0.1 | −0.1 | −0.1 | −0.1 | −0.1 | −0.1 | −0.2 | −0.2 | −0.2 | −0.3 | −0.1 |
| 4203.730 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | −0.1 | −0.1 |
| 4207.566 | −0.1 | 0.0 | 0.0 | −0.1 | +0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | −0.1 | −0.2 | −0.1 |
| 4216.136 | −0.1 | −0.1 | 0.0 | 0.0 | 0.0 | −0.2 | 0.0 | −0.2 | −0.1 | −0.1 | −0.2 | −0.3 | −0.4 |
| 4220.509 | 0.0 | +0.1 | 0.0 | 0.0 | +0.1 | 0.0 | +0.1 | 0.0 | +0.1 | 0.0 | +0.1 | +0.1 | −0.2 |
| 4232.887 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | 0.0 | +0.1 | 0.0 | −0.1 | 0.0 | 0.0 | −0.3 |
| 4233.328 | −0.1 | +0.1 | 0.0 | 0.0 | +0.1 | −0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | −0.1 | −0.1 |
| 4257.815 | +0.1 | +0.1 | 0.0 | +0.1 | +0.1 | +0.1 | +0.1 | +0.1 | 0.0 | +0.1 | +0.1 | +0.2 | +0.3 |
| 4258.477 | +0.1 | 0.0 | +0.1 | 0.0 | 0.0 | +0.1 | 0.0 | +0.1 | 0.0 | 0.0 | 0.0 | +0.1 | −0.2 |
| 4265.418 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +0.2 |
| 4266.081 | +0.1 | +0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | 0.0 | +0.1 | +0.2 | +0.3 |
| 4268.915 | +0.1 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | +0.1 | 0.0 | +0.1 | +0.2 | +0.2 | +0.2 |
| 4276.836 | 0.0 | 0.0 | +0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | +0.1 | +0.1 |
| 4283.169 | +0.1 | 0.0 | 0.0 | 0.0 | −0.1 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | +0.1 | +0.1 |
| 4284.838 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | −0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +0.3 | 0.0 |
| 4287.566 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | +0.1 |
| 4288.310 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | +0.3 |
| 4289.525 | +0.1 | 0.0 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | +0.1 | +0.1 | 0.0 | +0.1 | 0.0 | +0.3 |
| 4290.377 | 0.0 | −0.1 | −0.1 | 0.0 | −0.1 | 0.0 | 0.0 | −0.1 | −0.1 | −0.1 | 0.0 | −0.1 | −0.1 |
| 4290.542 | 0.0 | 0.0 | 0.0 | +0.1 | 0.0 | +0.1 | +0.1 | +0.1 | +0.1 | +0.1 | +0.1 | +0.2 | −0.1 |
| 4291.630 | +0.1 | +0.1 | 0.0 | 0.0 | 0.0 | +0.1 | +0.1 | +0.1 | +0.1 | +0.1 | +0.1 | +0.2 | +0.2 |

In Table IV the values at 79°.2 are based upon only seven plates, and hence are entitled to about one-half the weight of the other determinations.

An inspection of these results shows that the lines $\lambda$ 4196, $\lambda$ 4197, $\lambda$ 4216, and $\lambda$ 4290.38 give systematically low values of $\xi$, while $\lambda$ 4257, $\lambda$ 4266, $\lambda$ 4291, and possibly $\lambda$ 4268 give high values. A similar result was found for the first six of these lines in 1906–1907, and the general agreement of the two sets of determinations is sufficient to make the reality of these differences very probable, particularly in the case of the first three lines for which the differences are largest. Both series of observations give in the lower latitudes a value of $-0°.12$ for the lanthanum line $\lambda$ 4196, and $-0°.10$ for the mean of the two cyanogen lines $\lambda$ 4197 and $\lambda$ 4216. Since these lines originate at a low level in the solar atmosphere they furnish additional evidence for the conclusion based on the special studies of $H\alpha$ and $\lambda$ 4227 of calcium that the rate of angular rotation increases with increase of height above the sun's photosphere.

A comparison of the present results with those of 1906–1907 shows, however, that there is no such marked increase toward higher latitudes in the size of the deviations for these lines from the mean as was indicated by the earlier observations, although there is still some tendency in this direction. In the previous paper I ascribed this mainly to the effect of errors of measurement on the value of the angular velocity, an effect which in the higher latitudes becomes very large. At 75°, for example, a difference of 0.01 km in linear velocity corresponds to 0°.27 in the daily angular motion, and at 80° to 0°.41. There is still sufficient evidence, however, to warrant the presumption that the retardation becomes somewhat greater in higher latitudes, and this seems to be in harmony with the conclusions derived from investigations upon $H\alpha$ and $\lambda$ 4227 of calcium.

Of the other lines which give systematic deviations, $\lambda$ 4257, $\lambda$ 4266, and $\lambda$ 4290.38 gave similar results in the observations of 1906–1907, although the values were as a rule somewhat larger than those found here. The line $\lambda$ 4290.38 was referred to at that time as of particular interest, on account of being an "enhanced" line of titanium, and the additional evidence confirming its behavior is especially important. The remaining two lines $\lambda$ 4268 and $\lambda$ 4291 showed no

marked tendency toward large residuals, although giving small values of the same sign as those found above. The line λ 4291 is much strengthened at the limb of the sun.

The cyanogen line λ 4207, though showing a marked tendency toward negative residuals, gives considerably smaller values than do the other two cyanogen lines λ 4197 and λ 4216. This line is the most difficult of measurement of any in the list, and a part of the discrepancy may be due to this cause. In Rowland's general table it is ascribed to *Fe*, but in the table of corrections this is changed to *C*. The appearance of the line seems to favor a double origin, its breadth being considerably greater than that of either λ 4197 or λ 4216. If it is due in part to *Fe* the discrepancy would be fully explained.

The "enhanced" line of *Fe* λ 4233.33 gives a value which in the mean agrees exactly with the average from all of the lines. Like the "enhanced" line of *Ti* λ 4290.38, this line shows a considerable shift in position at the limb of the sun as compared with the center, which points to a relatively low-level origin for its "center of gravity." This result is in agreement with the rotation values. As opposed to this, however, is the occurrence of both of these lines in the chromosphere, the line λ 4233 being one of the most prominent in this region of the spectrum. The study of "enhanced" lines under pressure is likely to throw some light upon the behavior of these lines in the sun, but at present the contradiction seems to be marked.

Two other lines in the list deserve especial comment. These are λ 4283 and λ 4289, both due to calcium. The results indicate that slightly larger angular velocities are given by these lines, but the average difference is small, and the agreement with the result for the general reversing layer is close. We know from other considerations that the height attained in the sun's atmosphere by the gas giving rise to λ 4227 is very much greater than by that which produces most of the calcium lines of moderate intensity. Accordingly we should expect a marked difference in velocity. We are obliged, however, to draw a similar conclusion in regard to the two lines in the less refrangible part of the spectrum investigated by M. Perot by interference methods.[1] These lines, λ 5350 and λ 6122, were found to show very much greater angular velocities than the general reversing

[1] *Comptes Rendus*, **147**, 340, 1908.

layer and but slight equatorial acceleration. This result agrees much more closely with what I have found in the study of $\lambda$ 4227, a discussion of which will be given at a later point in this paper. The line $\lambda$ 6122 is one of the strong lines belonging to the red triplet in the second sub-series of calcium. The other lines do not belong to known series, although both $\lambda$ 4283 and $\lambda$ 4289 have certain definite relations with other lines in their vicinity. The three lines of shorter wavelength do not seem to be present in the chromosphere, at least in any considerable intensity, but $\lambda$ 6122 is given by Young in the revision of his chromospheric list.[1]

We may now pass on to a consideration of the general results. If mean values are formed from Table II and grouped about thirteen points of latitude we obtain a summary of the following form. The weights are given according to the number of observations.

TABLE V

| $\phi$ | Weight | $v$ km | $\xi$ | Period, Days |
|---|---|---|---|---|
| 0°3 | 21 | 2.063 | 14°65 | 24.57 |
| 4.1 | 15 | 2.040 | 14.52 | 24.79 |
| 11.2 | 12 | 1.992 | 14.44 | 24.98 |
| 14.9 | 18 | 1.944 | 14.28 | 25.21 |
| 19.2 | 14 | 1.868 | 14.04 | 25.64 |
| 29.7 | 16 | 1.671 | 13.66 | 26.35 |
| 34.1 | 15 | 1.557 | 13.39 | 26.97 |
| 44.7 | 17 | 1.283 | 12.81 | 28.10 |
| 49.6 | 16 | 1.141 | 12.49 | 28.82 |
| 60.0 | 22 | 0.811 | 11.52 | 31.25 |
| 65.0 | 17 | 0.678 | 11.41 | 31.55 |
| 74.9 | 17 | 0.398 | 10.84 | 33.21 |
| 79.2 | 7 | 0.274 | 10.29 | 35.26 |

The velocities given in this table have been plotted graphically, and the full line shown in Fig. 1 indicates the curve to which they correspond most closely. The general agreement of the points is good, the largest deviation from the curve falling at the point of lowest weight 79°2. Near 50° there is a distinct point of inflection. The curve shown by the broken line in the same figure represents the observations of 1906-1907. For numerical comparison the following table, giving the values from the two series of observations for every 5° of latitude, will be found useful. They are taken from large-scale

[1] Scheiner's *Astronomical Spectroscopy* (Frost), p. 422.

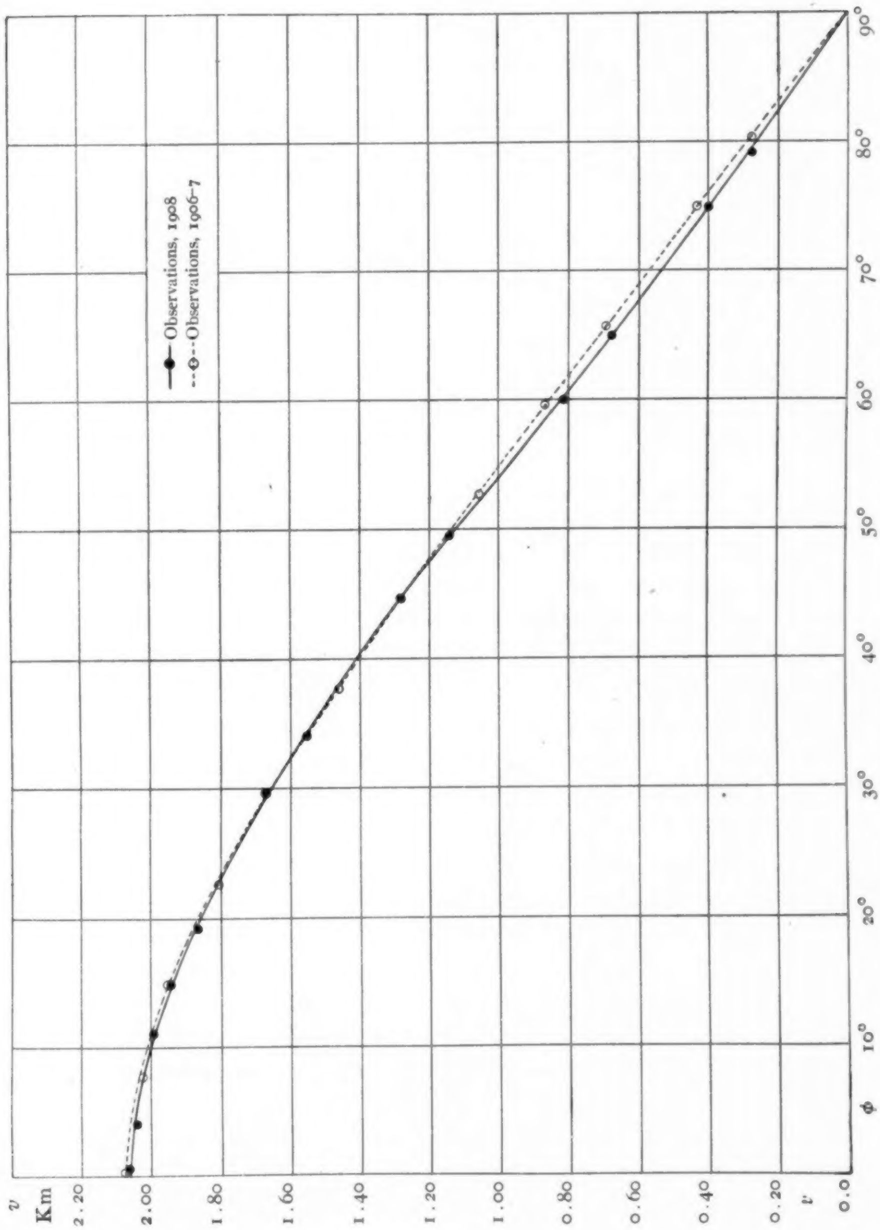



FIG. 1.—Radial Velocity Curves for the Reversing Layer in 1908 and 1906–1907.

drawings of the two curves, and are quite accurate enough for the purpose.

TABLE VI

| $\phi$ | $v$ (1908) | $v$ (1906–7) | $\xi$ (1908) | $\xi$ (1906–7) |
|---|---|---|---|---|
| | km | km | | |
| 0° | 2.060 | 2.077 | 14°.63 | 14°.75 |
| 5 | 2.043 | 2.056 | 14.56 | 14.65 |
| 10 | 2.000 | 2.012 | 14.42 | 14.50 |
| 15 | 1.943 | 1.954 | 14.28 | 14.36 |
| 20 | 1.863 | 1.872 | 14.08 | 13.14 |
| 25 | 1.761 | 1.768 | 13.80 | 13.85 |
| 30 | 1.659 | 1.660 | 13.60 | 13.61 |
| 35 | 1.537 | 1.532 | 13.32 | 13.28 |
| 40 | 1.414 | 1.406 | 13.10 | 13.03 |
| 45 | 1.274 | 1.274 | 12.79 | 12.79 |
| 50 | 1.126 | 1.137 | 12.44 | 12.56 |
| 55 | 0.967 | 0.991 | 11.97 | 12.27 |
| 60 | 0.820 | 0.851 | 11.64 | 12.08 |
| 65 | 0.677 | 0.711 | 11.37 | 11.94 |
| 70 | 0.533 | 0.569 | 11.06 | 11.81 |
| 75 | 0.396 | 0.429 | 10.86 | 11.77 |
| 80 | 0.266 | 0.290 | 10.88 | 11.85 |

A comparison of the two sets of observations shows slightly larger values for the earlier series between latitudes 0° and 25°, practical coincidence between 25° and 50°, and decidedly larger values again for the earlier series above this point. The differences in the lower latitudes reach a maximum of 0.017 km at the equator, and have an average value of about 0.01 km throughout the first 25° of latitude. These quantities, though larger than would be expected from errors of measurement, are still so small as to give little indication of the presence of systematic errors, or to show a possible variation in the sun's rate of rotation.

The matter is different in the higher latitudes, however. At 50° the two series of observations begin to separate from one another, and at 70° a difference of 0.036 km is reached. Though determinations of the probable error have shown that the value is usually somewhat larger in higher latitudes where the measured displacements are small, differences of this amount are evidently systematic, and the cause is to be sought either in a real change of rotation period in the higher latitudes, or in some source of systematic error in the observations, whether instrumental, or in the sun itself. At present there seems to be no definite criterion for settling the question, but there is, I

think, sufficient evidence to furnish a presumption against the reality of a change in the sun's rotation period. The most important part of this is the difficulty of conceiving a change of rotation rate which is confined to high latitudes. It would certainly seem probable that any such change of rate would be in some way connected with the sun-spot activity, and would be conspicuous in the zones of greatest spot frequency between 10° and 30° of latitude. This is the conclusion which Halm has drawn from his two series of observations,[1] which show large differences in these zones as well as in higher latitudes. The results given here indicate almost no difference between the values of 1908 and those of 1906–1907 for these zones.

As against this it may be argued that evidence is gradually accumulating to show that the rotation rate at the sun's equator is more nearly constant than at any other latitude, and that the so-called "equatorial acceleration" is more truly a polar retardation. In this direction, for example, point the observations of Halm showing the greatest variations in angular velocity in higher latitudes, as well as the results given here, on the one hand for the low-level lines of lanthanum and cyanogen, and on the other hand for the high-level line $\lambda$ 4227 of calcium and the $\alpha$ line of hydrogen. All of these show the greatest differences in angular velocity toward the pole. The force of this argument is considerable, and it may perhaps account for the apparent anomaly of a change in rotation rate which is confined to high latitudes on the sun.

A second argument which is entitled to considerable weight is the satisfactory agreement of the results of the 1908 observations with Faye's formula for the rotation period of sun-spots, and the fact that Dunér's observations, as well as those of Halm, are also in excellent accord with the same formula. If we put

$$v=(a+b\cos^2\phi)\cos\phi\ ,$$
$$\xi=a'+b'\cos^2\phi\ ,$$

and deduce the values of the constants by solving the normal equations given by the observations, we find

$$v=(1.506+0.549\cos^2\phi)\cos\phi\ ,$$
$$\xi=10^\circ\!.62+3^\circ\!.99\cos^2\phi\ .$$

[1] *Transactions of the Royal Society of Edinburgh*, **41**, Part I.

In obtaining these results the observations have been assigned their respective weights, but the observation equations have been formed by multiplying with these weights instead of by their square roots.

The residuals given by these equations are as follows:

TABLE VII

| $\phi$ | $v$ km | $\xi$ |
|---|---|---|
| 0°.3 | +0.009 | +0°.04 |
| 4.1 | −0.006 | −0.07 |
| 11.2 | −0.003 | −0.02 |
| 14.9 | −0.007 | −0.07 |
| 19.2 | −0.016 | −0.14 |
| 29.7 | +0.003 | +0.03 |
| 34.1 | 0.000 | +0.03 |
| 44.7 | +0.016 | +0.17 |
| 49.6 | +0.015 | +0.19 |
| 60.0 | −0.010 | −0.10 |
| 65.0 | +0.001 | +0.08 |
| 74.9 | −0.004 | −0.05 |
| 79.2 | −0.013 | −0.47 |

The general agreement of these results is quite as satisfactory as could be expected from a formula of such simple form, the largest residual being the point of low weight and high latitude at 79°.2. In the 1906–1907 observations, on the other hand, it was found that Faye's equation gave deviations which, though small, were systematically negative in mean latitudes, and positive in high latitudes, and an equation containing three constants was employed in order to obtain suitable agreement. The fact that the results of Dunér and Halm as well as my own are satisfactorily represented by Faye's formula, while Spörer's formula gives systematic deviations, points to the conclusion that this simple expression represents with sufficient accuracy the motion of the sun's reversing layer to within about 10° of the pole, beyond which observations are lacking at present. The investigation of the zone between 80° and 90° of latitude, undertaken when the position of the sun's pole is most favorable for observations necessarily so difficult, will be of great interest, and should increase materially our knowledge of the law of rotation.

The close absolute agreement of Dunér's results with those given by the 1908 observations is also opposed to the idea of a change in the sun's rate of rotation. A comparison for the six latitudes employed by Dunér gives the following summary:

TABLE VIII

| $\phi$ | Dunér | Adams |
|---|---|---|
| | km | km |
| 0°4 | 2.07 | 2.06 |
| 15.0 | 1.97 | 1.94 |
| 30.0 | 1.69 | 1.67 |
| 45.0 | 1.27 | 1.27 |
| 60.0 | 0.80 | 0.81 |
| 74.9 | 0.40 | 0.40 |

Dunér's results are based upon observations extending through six years, from 1887 to 1889, and from 1899 to 1901. Halm in his discussion[1] advocating a variation in the rotation rate of the sun has analyzed these observations, using as a basis Faye's equation in the form

$$v=(a-b\sin^2\phi)\cos\phi\ .$$

In this form the 1908 observations would give

$$v=(2.054-0.549\sin^2\phi)\cos\phi\ .$$

Halm's comparison of the Upsala and the Edinburgh observations on the basis of a three-year period would require a minimum value of the quantity $a$ for the epoch 1908.5, which is about the mean date of these observations. The value found here, however, is practically equal to the largest value obtained by Halm in any one of the years 1901 to 1906, which included two maximum values of $a$. The value of $b$ is also opposed to the idea of a progressive change in this quantity unless we assume that after decreasing from 1901 to 1905, it has reversed its direction and returned in 1908 to a value almost identical with that of 1901.

The general conclusion from these considerations seems to be that it is probable that the observations of 1906–1907 were affected by slight systematic errors which amounted to as much as 0.03 km in the higher latitudes. Attention has already been called to the fact that the astigmatism of the sun's image may have influenced the values to some extent, and it is possible that changes of focus may have also introduced slight errors. Another possible source of error due to disturbances on the sun itself will be discussed in a succeeding paragraph. In any event, it seems probable that the 1908 series of observations, being comparatively free from such defects of the solar

[1] *Astronomische Nachrichten*, **173**, 294, 1907.

image, and containing as they do a valuable check upon instrumental conditions in the exposures upon the pole of the sun, deserve a considerably higher degree of confidence.

Determinations of the probable errors for the observations of 1908 indicate that these are smaller than for the earlier series. From a number of plates taken at random we find at 45° of latitude: for a single line,

$$\epsilon = \pm 0.009 \text{ km} ,$$

or for the mean value from the plate,

$$\epsilon_0 = \pm 0.002 \text{ km} .$$

The 1906–1907 observations gave

$$\epsilon = \pm 0.015 \text{ km} ,$$

and

$$\epsilon_0 = \pm 0.004 \text{ km} .$$

In these determinations the lines giving the principal systematically large or small values have been omitted. The 1908 probable errors, accordingly, are based upon sixteen lines, and the 1906–1907 errors upon fourteen lines. The probable errors are slightly larger in the higher latitudes.

### THE MOTION OF THE REVERSING LAYER IN THE VICINITY OF A SOLAR VORTEX

On September 15 four plates of the region of the spectrum including $\lambda$ 4227 were made with the rotation apparatus at latitudes ranging from 0° to 75° in steps of 15° each. On this date two spots of considerable size, one at 6° south latitude, the other at 11° north latitude, were close to the west limb of the sun, the northern spot being but a few hours distant from the visible edge, while the other had just passed beyond it. Plates taken with the spectroheliograph had shown these spots to be surrounded by vortices in which the motion was apparently in opposite directions in the two cases. Observations made by Hale during the passage of the spots across the sun's disk also showed opposite directions of polarization for the components of the double lines in the spot spectra. The region between the two spots, as indicated by photographs taken with the $H\alpha$ line, was in an extremely chaotic state, owing probably to the intermingling of the vortices. One of the settings in the case of the rotation plates fell

at latitude 14°.9 north, or 4° north of one of the spots; the other at 0°, or 6° north of the second spot, in this case being considerably east as well. When the measurement of the plates was begun it was at once seen that these two latitudes gave extremely discordant values as compared with the normal. Accordingly, the plates were investigated separately and measures made not only on $\lambda$ 4227, but also on a number of the general reversing layer lines. As in the case of all of the photographs intended for the study of $\lambda$ 4227, the density was made great in order to facilitate settings upon this very broad line, and for this reason the list of lines regularly used for the reversing layer could not be employed. So a number of stronger lines which appeared suitable for measurement were selected. The individual results for these lines and for $\lambda$ 4227 are given in the following table:

TABLE IX

| Number of Plate | Date 1908 | Number of Lines | $\phi$ | Reversing Layer $v$ km | $\lambda$ 4227 $v$ km |
|---|---|---|---|---|---|
| $\omega$ 173 | Sept. 15 | 12 | 0°.0 | 1.918 | 1.974 |
| | | | 14.9 | 1.896 | 1.960 |
| | | | 29.8 | 1.662 | 1.733 |
| | | | 44.7 | 1.249 | 1.325 |
| | | | 62.8 | 0.722 | 0.813 |
| | | | 75.3 | 0.387 | 0.492 |
| $\omega$ 174 | Sept. 15 | 12 | 0.0 | 1.900 | 1.963 |
| | | | 14.9 | 1.882 | 1.959 |
| | | | 29.8 | 1.673 | 1.723 |
| | | | 44.7 | 1.234 | 1.325 |
| | | | 62.8 | 0.731 | 0.808 |
| | | | 75.3 | 0.405 | 0.460 |
| $\omega$ 175 | Sept. 15 | 12 | 0.0 | 1.954 | 2.062 |
| | | | 14.9 | 1.913 | 1.984 |
| | | | 29.8 | 1.664 | 1.722 |
| | | | 44.7 | 1.257 | 1.282 |
| | | | 62.8 | 0.717 | 0.815 |
| | | | 75.3 | 0.377 | 0.460 |
| $\omega$ 176 | Sept. 15 | 12 | 0.0 | 1.949 | 2.029 |
| | | | 14.9 | 1.906 | 1.982 |
| | | | 29.8 | 1.667 | 1.722 |
| | | | 44.7 | 1.222 | 1.303 |
| | | | 62.8 | 0.726 | 0.859 |
| | | | 75.3 | 0.287 | 0.493 |

If we combine the preceding results we obtain the following summary. The normal value for the reversing layer is given in the third column for comparison.

TABLE X

| $\phi$ | $v$ km | Normal $v$ km | $\lambda$ 4227 $v$ km |
|---|---|---|---|
| 0°0 | 1.930 | 2.060 | 2.007 |
| 14.9 | 1.899 | 1.945 | 1.971 |
| 29.8 | 1.666 | 1.662 | 1.725 |
| 44.7 | 1.240 | 1.277 | 1.309 |
| 62.8 | 0.724 | 0.720 | 0.844 |
| 75.3 | 0.389 | 0.393 | 0.476 |

It is seen that in latitudes 30° and more the values derived from these plates agree well with the normal values. At 15°, however, the results are decidedly lower both for the reversing layer and for $\lambda$ 4227. The same is true of latitude 0°, and the differences here are even greater. The plates taken with the spectroheliograph indicate a direction of motion in the vortex surrounding the northern spot that is counter-clockwise as seen from above. Accordingly, for a position north of the spot the component of motion in the line of sight at the sun's west limb would be toward the observer, and its effect would be to reduce the value of the linear velocity observed on the rotation plates. This is what is found.

In the case of the second spot the vortical motion should be in the opposite direction, judging from polarization and spectroheliograph results, and this is opposed to what is found from the rotation results at latitude 0°, if the latter are affected by this vortex. An examination of the *Hα* photographs shows, however, that the point from which the light is taken into the rotation apparatus falls at a considerable distance from the spot, and over a region of great irregularity where the two vortices seem to mix with each other, and where no well-defined direction of motion seems to exist. In any case the observations should be much more extensive and should include more points of latitude in order to give any definitive evidence as to the question of direction of rotation in such regions. The important fact is that solar vortices of this nature seem to have a marked influence upon the reversing layer, and that rotation results obtained when such regions are at the limb of the sun may be very seriously affected by the proper motion of the reversing layer. Since these vortices usually accompany spots, a natural inference would be that the greatest variations in values of the rotational velocity would be found in the latitudes where

spots are most frequent, that is, in the zone 10° to 30°. It is possible that this may to some extent influence the results found by Halm in the two series 1901–1902 and 1903. The fact that vortices frequently exist where there are no spots, and extend to high latitudes, would also account for variations of rotational velocity found beyond the zones of principal spot activity. This may be a partial cause of the differences in latitudes 50° to 80° between the results 1906–1907 and 1908, since neither series contains a sufficiently large number of observations to eliminate the influence of a few cases of such proper motion. The necessary conclusion is that in making spectroscopic observations of the rotation of the sun especial care should be taken to avoid regions of the surface which show evidences of vortical disturbance.

### RESULTS FOR $\lambda$ 4227

The study of at least one of the stronger lines of calcium was made most desirable by the results obtained for the *a* line of hydrogen, as well as the differences found among the various lines used in the investigation of the reversing layer. Unfortunately the lines of greatest interest, namely H and K, are practically excluded by the immense variation in their physical appearance at different parts of the sun's disk, according to the presence or absence of calcium flocculi. They also appear to be especially subject to disturbances arising from motion of the calcium vapor in the line of sight.[1] Excluding these lines, it was evident that the so-called "blue line" of calcium at $\lambda$ 4226.91 would prove most valuable for investigation. It is known to rise to a great height in the chromosphere, although, of course, much lower than either H or K, and its appearance in the spectrum is such as to admit of measures of considerable accuracy. There is also a decided practical advantage in having the line fall within the region observed for the motion of the reversing layer.

It at first seemed probable that the same plates which were used for the study of the reversing layer could be employed for $\lambda$ 4227 as well. A few measures, however, showed that the line is of such great intensity, and the wings so broad under high dispersion that especially dense negatives would have to be employed, and it is upon

[1] *Contributions from the Solar Observatory*, No. 6; *Astrophysical Journal*, **23**, 45–53, 1906.

a series of plates taken for this line alone that the results given here are based. An interesting by-product of the study of the line is the discovery that it is double. Several plates show this fact clearly, although the actual separation is extremely small.

The results for the individual plates are given in the table below. In each case the value is the mean of two measures, one by Miss Lasby, and the other by myself.

TABLE XI

| Plate | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\omega$ 149 | 0°3 | 2.10 | 14°6 | 2.01 | 29°6 | 1.75 | 44°9 | 1.32 | 60°1 | 0.84 | 74°9 | 0.47 |
| $\omega$ 150 | 0.3 | 2.09 | 14.6 | 1.99 | 29.6 | 1.72 | 44.9 | 1.32 | 60.1 | 0.83 | 74.9 | 0.46 |
| $\omega$ 152 | 0.1 | 2.14 | 14.6 | 1.99 | 29.7 | 1.72 | 44.6 | 1.32 | 59.8 | 0.89 | 74.9 | 0.47 |
| $\omega$ 153 | 0.1 | 2.12 | 14.6 | 1.99 | 29.7 | 1.74 | 44.6 | 1.33 | 59.8 | 0.85 | 74.9 | 0.43 |
| $\omega$ 154 | 0.1 | 2.10 | 14.6 | 2.02 | 29.7 | 1.74 | 44.6 | 1.34 | 59.8 | 0.81 | 74.9 | 0.43 |
| $\omega$ 157 | 0.3 | 2.11 | 14.6 | 2.04 | 29.5 | 1.71 | 44.1 | 1.36 | 58.4 | 0.88 | 72.0 | 0.50 |
| $\omega$ 158 | 0.3 | 2.10 | 14.6 | 2.04 | 29.5 | 1.70 | 44.1 | 1.34 | 58.4 | 0.88 | 72.0 | 0.48 |
| $\omega$ 167 | 0.4 | 2.15 | 14.5 | 2.05 | 29.4 | 1.78 | 44.3 | 1.40 | 59.5 | 0.92 | 74.4 | 0.52 |
| $\omega$ 168 | 0.4 | 2.12 | 14.5 | 2.05 | 29.4 | 1.74 | 44.3 | 1.39 | 59.5 | 0.92 | 74.4 | 0.57 |
| $\omega$ 169 | 0.4 | 2.15 | 14.5 | 2.06 | 29.4 | 1.74 | 44.3 | 1.39 | 59.5 | 0.89 | 74.4 | 0.57 |
| $\omega$ 170 | 0.4 | 2.16 | 14.5 | 2.05 | 29.4 | 1.75 | 44.3 | 1.42 | 59.5 | 0.97 | 74.4 | 0.54 |
| $\omega$ 188 | 0.1 | 2.10 | 14.8 | 2.09 | 29.7 | 1.78 | 44.9 | 1.36 | 60.2 | 0.90 | 75.9 | 0.48 |
| $\omega$ 189 | 0.1 | 2.12 | 14.8 | 2.02 | 29.7 | 1.78 | 44.9 | 1.39 | 60.2 | 0.94 | 75.9 | 0.48 |

These values give the following summary:

TABLE XII

| $\phi$ | $\lambda$ 4227 | | | Reversing Layer | | |
|---|---|---|---|---|---|---|
| | $v$ km | $\xi$ | Period | $v$ km | $\xi$ | Period |
| 0°2 | 2.12 | 15°0 | 23.9 | 2.06 | 14°7 | 24.6 |
| 14.6 | 2.03 | 14.9 | 23.2 | 1.95 | 14.3 | 25.2 |
| 29.6 | 1.74 | 14.2 | 25.3 | 1.67 | 13.7 | 26.4 |
| 44.6 | 1.36 | 13.6 | 26.5 | 1.28 | 12.8 | 28.1 |
| 59.6 | 0.89 | 12.5 | 28.8 | 0.82 | 11.5 | 31.2 |
| 74.6 | 0.49 | 13.1 | 27.4 | 0.40 | 10.8 | 33.2 |

A comparison of the results for $\lambda$ 4227 with those for the reversing layer shows: first, that the absolute velocity values are larger for $\lambda$ 4227; and second, that the decrease of angular velocity is much less marked toward higher latitudes. At the equator the difference in angular velocity is 0°4, while at 45° of latitude it is 0°8. The sudden increase in the value of $\xi$ at 75° will be discussed more fully in connection with the results for *Hα*. If we assume, however, that the increase is due to accidental errors, and take values from the curve

at 60° and 75°, we find at 60° a difference in the value of $\xi$ from that for the reversing layer of 1°4, and at 75° of 1°5. As will be seen later, similar results are found for *Hα*.

Reference has already been made to the results obtained by M. Perot for two of the less refrangible lines of calcium by the aid of interference methods. Preliminary values of the angular velocity given by him are as follows:

TABLE XIII

| $\lambda$ | $\phi=0°$ | $\phi=45°7$ |
|---|---|---|
| 5349.6 | 15°1 | 14°2 |
| 6122.4 | 14.7 | 14.2 |

At the equator these results agree closely with those for $\lambda$ 4227. At 45°, however, they give a considerably larger value, indicating less equatorial acceleration than in the case of $\lambda$ 4227. This difference may be much modified by more complete results, but if it still remains it will indicate a higher effective level for these lines at the limb than for the more refrangible line. At first sight, in view of the great strength of $\lambda$ 4227 in the chromosphere, this would seem improbable, but the width and the intensity of its wings and the comparative narrowness of the central line indicate that a large part of the line is produced in a region of relatively dense calcium vapor, and so at a moderately low level. This level must, nevertheless, be above that in which the wings of the stronger iron lines in the violet are formed, since plates of the center and limb spectrum show but a slight effect for $\lambda$ 4227 when the wings of the iron lines are almost completely obliterated. This is in agreement with the rotation values.

## RESULTS FOR THE *α* LINE OF HYDROGEN

The special investigation of the *Hα* line was begun, as stated in a previous paper,[1] in consequence of the remarkable behavior of this line at the limb of the sun. Its marked increase in width, and the absence of any displacement such as is found for the great majority of lines between limb and center, pointed to a very high-level origin, a fact, of course, well known from observations of the intensity of the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 24; *Astrophysical Journal*, **27**, 213–218, 1908.

line in the upper chromosphere. Measures of the center and limb plates also indicated a considerably higher rotational velocity, and this result was confirmed by the earliest rotation plates. The preliminary values for a few of these plates were published in the paper already cited. At the same time at which these determinations were being made, measures were obtained of the motion of rotation given by the hydrogen flocculi upon plates taken with the spectroheliograph in the $\delta$ line of hydrogen.[1] These showed a marked tendency toward a uniform rate of rotation, but the absolute rate was decidedly smaller than that derived from the spectroscopic measures. Two possible explanations of this behavior at once presented themselves. The first was that $H\delta$ might give a different rate from $H\alpha$, a conclusion by no means improbable, in view of the different way in which they are affected at the limb of the sun. The second was that the marked increase of intensity of $H\alpha$ near the limb might indicate quite a different level for the spectroscopic determinations from that of the hydrogen flocculi. Photographs of the $H\alpha$ flocculi were essential to test either hypothesis, and experiments were begun by Mr. Hale and myself with the 30-foot spectroheliograph. These when continued by Hale and Ellerman with the 5-foot spectroheliograph led to the discovery of the solar vortices.[2]

The earliest photographs of the $H\alpha$ line were taken on rapid plates sensitized to the red with pinacyanol. After the publication of Wallace's sensitizing process for "Pan-iso" plates[3] this was employed to great advantage. It was found, moreover, that the exposure times on rapid plates sensitized in this way were very moderate. Accordingly it occurred to me to endeavor to obtain the advantage of finer silver grain by the use of slower plates, and for this purpose the Seed "Process" plates were tried. These have proved very successful, giving fine grain and excellent contrast, while the ratio of sensitiveness in the red to that of similarly treated rapid plates seems to be greater than the corresponding ratio of the unbathed plates in the blue. The advantages in the use of these plates for such a line as

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 25; *Astrophysical Journal*, **27**, 219–229, 1908.

[2] Hale, *Contributions from the Mount Wilson Solar Observatory*, No. 26; *Astrophysical Journal*, **28**, 100, 1908.

[3] *Astrophysical Journal*, **26**, 299–325, 1907.

$H\alpha$ are considerable, since the edges of the line are at best poorly defined, and the finer grain and the superior contrast assist materially in the measurement of the line. It is probable that the later plates show a considerably higher degree of accuracy for this reason, and in the summaries of the results which follow, each series begins with the first photographs taken on the slower plates.

It was found early in the study of the $H\alpha$ line that its width changes very rapidly within a small distance of the sun's limb. In fact, the entire variation in appearance between limb and center seems to take place within a distance of little more than one-thirtieth of the solar radius. It seemed probable that this effect might be due to the relative level in the two cases, the effective level of the line at the limb being higher than that inside the limb. If this is the case, we should expect the effect to show in the rotation values given by the line in the two positions. Accordingly two series of observations have been made on $H\alpha$; the first at points close to the limb; the second at points averaging about 3 mm inside the limb. The values obtained from the latter have, of course, been reduced to the limb by correcting for this distance.

The results for the individual plates of $H\alpha$ near the limb follow. As in the case of $\lambda$ 4227 each result is based on two measures.

TABLE XIV

| Plate | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\omega$ 110 | 0°2 | 2.12 | 15°1 | 2.04 | 29°9 | 1.75 | 44°3 | 1.45 | 60°4 | 0.97 | 75°2 | 0.53 |
| $\omega$ 115 | 0.4 | 2.13 | 14.6 | 2.03 | 29.6 | 1.72 | 44.9 | 1.38 | 60.5 | 0.94 | 75.9 | 0.49 |
| $\omega$ $118_1$ | 0.6 | 2.15 | 14.4 | 2.05 | 29.4 | 1.77 | 44.9 | 1.43 | 60.4 | 0.98 | 75.9 | 0.49 |
| $\omega$ $118_2$ | 0.6 | 2.17 | 14.4 | 2.06 | 29.4 | 1.74 | 44.9 | 1.44 | 60.4 | 0.99 | 75.9 | 0.52 |
| $\omega$ 122 | 0.5 | 2.13 | 14.5 | 1.99 | 29.5 | 1.79 | 44.5 | 1.35 | 59.5 | 0.93 | 75.1 | 0.49 |
| $\omega$ 123 | 0.5 | 2.16 | 14.5 | 2.03 | 29.5 | 1.78 | 44.5 | 1.41 | 59.5 | 0.96 | 75.1 | 0.48 |
| $\omega$ $124_1$ | 0.5 | 2.10 | 14.5 | 2.09 | 29.5 | 1.78 | 44.5 | 1.41 | 59.5 | 0.99 | 75.1 | 0.53 |
| $\omega$ $124_2$ | 0.5 | 2.14 | 14.5 | 2.05 | 29.5 | 1.78 | 44.5 | 1.39 | 59.5 | 0.97 | 75.1 | 0.52 |
| $\omega$ $125_1$ | 0.5 | 2.17 | 14.5 | 2.05 | 29.5 | 1.82 | 44.5 | 1.42 | 59.5 | 0.97 | 75.0 | 0.50 |
| $\omega$ $125_2$ | 0.5 | 2.14 | 14.5 | 2.02 | 29.5 | 1.80 | 44.5 | 1.43 | 59.5 | 0.96 | 75.0 | 0.49 |
| $\omega$ $126_1$ | 0.5 | 2.15 | 14.5 | 2.04 | 29.5 | 1.81 | 44.5 | 1.41 | 59.5 | 0.95 | 75.0 | 0.50 |
| $\omega$ $126_2$ | 0.5 | 2.16 | 14.5 | 2.02 | 29.5 | 1.84 | 44.5 | 1.42 | 59.5 | 0.98 | 75.0 | 0.53 |
| $\omega$ $127_1$ | 0.5 | 2.16 | 14.5 | 2.04 | 29.5 | 1.80 | 44.5 | 1.38 | 59.5 | 0.95 | 74.5 | 0.52 |
| $\omega$ $127_2$ | 0.5 | 2.17 | 14.5 | 2.02 | 29.5 | 1.83 | 44.5 | 1.40 | 59.5 | 0.98 | 74.5 | 0.50 |
| $\omega$ 129 | 0.5 | 2.15 | 14.5 | 2.04 | 29.5 | 1.82 | 44.5 | 1.41 | 59.5 | 0.96 | 74.5 | 0.52 |
| $\omega$ $130_1$ | 0.5 | 2.15 | 14.5 | 2.03 | 29.5 | 1.80 | 44.5 | 1.40 | 59.5 | 1.00 | 74.5 | 0.53 |
| $\omega$ $130_2$ | 0.5 | 2.14 | 14.5 | 2.03 | 29.5 | 1.79 | 44.5 | 1.40 | 59.5 | 0.95 | 74.5 | 0.55 |
| $\omega$ 131 | 0.5 | 2.14 | 14.5 | 2.04 | 29.5 | 1.81 | 44.5 | 1.41 | 59.5 | 0.98 | 74.5 | 0.52 |
| $\omega$ 141 | 0.1 | 2.14 | 14.8 | 2.04 | 29.7 | 1.79 | 44.9 | 1.41 | 60.1 | 0.99 | 75.1 | 0.57 |
| $\omega$ 144 | 0.1 | 2.15 | 14.8 | 2.04 | 29.7 | 1.79 | 44.9 | 1.44 | 60.1 | 1.00 | 75.1 | 0.59 |
| $\omega$ 171 | 0.3 | 2.12 | 15.2 | 2.04 | 30.0 | 1.76 | 45.0 | 1.42 | 60.2 | 0.98 | 75.0 | 0.60 |

If we form means from the above results we obtain the following summary. The corresponding results for the reversing layer are given in the last three columns.

TABLE XV

| $\phi$ | $H\alpha$ | | | Reversing Layer | | |
|---|---|---|---|---|---|---|
| | $v$ km | $\xi$ | Period | $v$ km | $\xi$ | Period |
| 0°4 | 2.15 | 15°2 | 23.6 | 2.06 | 14°6 | 24.6 |
| 14.6 | 2.05 | 15.0 | 24.1 | 1.95 | 14.3 | 25.2 |
| 29.6 | 1.79 | 14.6 | 24.7 | 1.67 | 13.7 | 26.3 |
| 44.6 | 1.41 | 14.0 | 25.7 | 1.28 | 12.8 | 28.1 |
| 59.8 | 0.97 | 13.7 | 26.2 | 0.81 | 11.5 | 31.2 |
| 75.0 | 0.52 | 14.3 | 25.2 | 0.40 | 10.8 | 33.2 |

Two important conclusions are evident from an inspection of these results. The first is that the hydrogen gas giving rise to $H\alpha$ moves at a much higher velocity than the reversing layer. The second is that the law of change of velocity with latitude is very different, the equatorial acceleration being comparatively slight. Both of these conclusions were derived from the preliminary values given in the paper previously referred to, but the absolute values differ considerably. In the preliminary results, for example, no equatorial acceleration whatever was found, while these results give a difference of over 1° in the value of $\xi$ between the equator and high latitudes. The absolute velocity at the equator was also found to be about 0°3 larger in the preliminary results. The greater part of this difference is no doubt due to the small number of plates upon which the early measures were based, and also to the fact that these plates were comparatively inferior in quality, being obtained previous to the use of the finer-grained "Process" plates. There is, however, another cause which may contribute to a systematic difference in the two cases. This is the fact that the earlier plates were taken at points closer to the edge of the sun than were the later plates, so close, indeed, that on two out of the three $H\alpha$ plates used in the early measures, the bright chromospheric line appears in addition to the dark line. Accordingly it is altogether probable, judging from results found for $H\alpha$ inside the limb, that in the plates used in the preliminary determination a higher average level was in question than in the case of the plates

discussed here, and that this higher level would give larger velocities and less equatorial acceleration.

The results obtained from plates of $H\alpha$ taken at points within the limb are given below.

TABLE XVI

| Plate | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km | $\phi$ | $v$ km |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\omega$ 116 | 0°1 | 2.14 | 14°9 | 2.03 | 29°9 | 1.72 | 45°1 | 1.38 | 60°0 | 0.91 | 76°2 | 0.44 |
| $\omega$ $119_1$ | 0.6 | 2.12 | 14.4 | 2.01 | 29.4 | 1.71 | 44.9 | 1.36 | 60.4 | 0.91 | 75.9 | 0.46 |
| $\omega$ $119_2$ | 0.6 | 2.13 | 14.4 | 2.01 | 29.4 | 1.74 | 44.9 | 1.32 | 60.4 | 0.88 | 75.9 | 0.42 |
| $\omega$ 131 | 0.8 | 2.13 | 14.4 | 2.01 | 29.4 | 1.76 | 44.9 | 1.36 | 59.2 | 0.96 | 75.9 | 0.50 |
| $\omega$ 137 | 0.5 | 2.10 | 14.5 | 1.99 | 29.4 | 1.73 | 44.6 | 1.39 | 59.8 | 0.88 | 74.9 | 0.49 |
| $\omega$ 138 | 0.4 | 2.08 | 14.5 | 2.00 | 29.4 | 1.74 | 44.6 | 1.36 | 59.8 | 0.87 | 74.9 | 0.49 |
| $\omega$ 142 | 0.1 | 2.14 | 14.8 | 2.03 | 29.7 | 1.75 | 44.9 | 1.41 | 60.1 | 0.94 | 75.1 | 0.50 |
| $\omega$ 143 | 0.1 | 2.13 | 14.8 | 1.99 | 29.7 | 1.71 | 44.9 | 1.37 | 60.1 | 0.94 | 75.1 | 0.52 |
| $\omega$ 172 | 0.3 | 2.08 | 15.2 | 1.97 | 30.0 | 1.71 | 45.0 | 1.37 | 60.2 | 0.92 | 75.0 | 0.53 |

Summarized, these results give the following. The values for $H\alpha$ at the limb are added for comparison.

TABLE XVII

| $\phi$ | $H\alpha$ (within limb) | | | $H\alpha$ (limb) | | |
|---|---|---|---|---|---|---|
| | $v$ km | $\xi$ | Period | $v$ km | $\xi$ | Period |
| 0°4 | 2.12 | 15°0 | 24.0 | 2.15 | 15°2 | 23.6 |
| 14.7 | 2.00 | 14.7 | 24.5 | 2.04 | 15.0 | 24.1 |
| 29.6 | 1.73 | 14.1 | 25.5 | 1.79 | 14.6 | 24.7 |
| 44.8 | 1.37 | 13.7 | 26.3 | 1.41 | 14.0 | 25.7 |
| 60.1 | 0.91 | 13.0 | 27.8 | 0.96 | 13.6 | 26.3 |
| 75.4 | 0.48 | 13.6 | 26.4 | 0.51 | 14.2 | 25.4 |

A comparison of these values shows not only that the velocities for $H\alpha$ at the limb are larger than those at points within the limb, but also that the equatorial acceleration is less, or the rate of rotation more nearly uniform. It is clear, then, that the hydrogen gas producing $H\alpha$ at the limb must lie at a generally higher level, a result which is in agreement with the character of the spectrum line in the two positions. In the case of a gas such as hydrogen, which extends to a great height in the solar atmosphere, it is evident that the thickness of the absorbing layer in the line of sight must increase very rapidly toward the sun's limb. The level of the effective absorption, accordingly, will in general be higher at the limb than on the disk of the sun, and if the upper regions are cooler the effect should show itself in the change of

intensity which accompanies decrease of temperature. Kayser has shown[1] that with decreasing temperature there is a strengthening of the red line of hydrogen relative to the more refrangible lines. At the limb of the sun it is found that the intensity of $H\alpha$ is greatly increased as compared with its intensity at the center, $H\beta$ is also slightly increased, while $H\gamma$ and $H\delta$ are somewhat weakened. The agreement with the laboratory results, then, is complete.

It is as yet impossible to compare these results with those obtained from the study of the $H\alpha$ flocculi on spectroheliograph plates, but measures now being made by Miss Ware will furnish the material for such a comparison. It will also be of special interest to discuss the results of the $H\alpha$ and the $H\delta$ flocculi separately, since the general evidence at present indicates a higher level for the $H\alpha$ flocculi as a class. It has not been found possible to use either $H\gamma$ or $H\delta$ to advantage in the spectroscopic study, both on account of the inherent character of the lines and the disturbances produced by the presence of foreign lines.

The results for $\lambda$ 4227, $H\alpha$ at the limb and $H\alpha$ within the limb, are shown graphically in Fig. 2. The fourth curve, indicated by a dotted line, corresponds to the linear velocity of a body rotating with the uniform angular velocity of 15°2 a day. It is, of course, the usual cosine curve. A marked feature of all of the results is the sudden increase in angular velocity near 75° of latitude. Although at this latitude the angular velocity is especially sensitive to small differences in linear velocity, the persistence of the effect for the three independent series of observations furnishes some presumption of its reality. It is, moreover, the counterpart of the differences from the general reversing layer found among the low-level lines. These, as was seen from the results given earlier in this paper, show exceptionally large deviations in the higher latitudes. At present the question of the reality of this effect must remain an open one.

## GENERAL SUMMARY

The main results of this investigation may be summarized as follows:

1. Observations of the rotation of the sun during 1908 give values

[1] *Festschrift* Ludwig Boltzmann (Leipzig, 1904), p. 38.

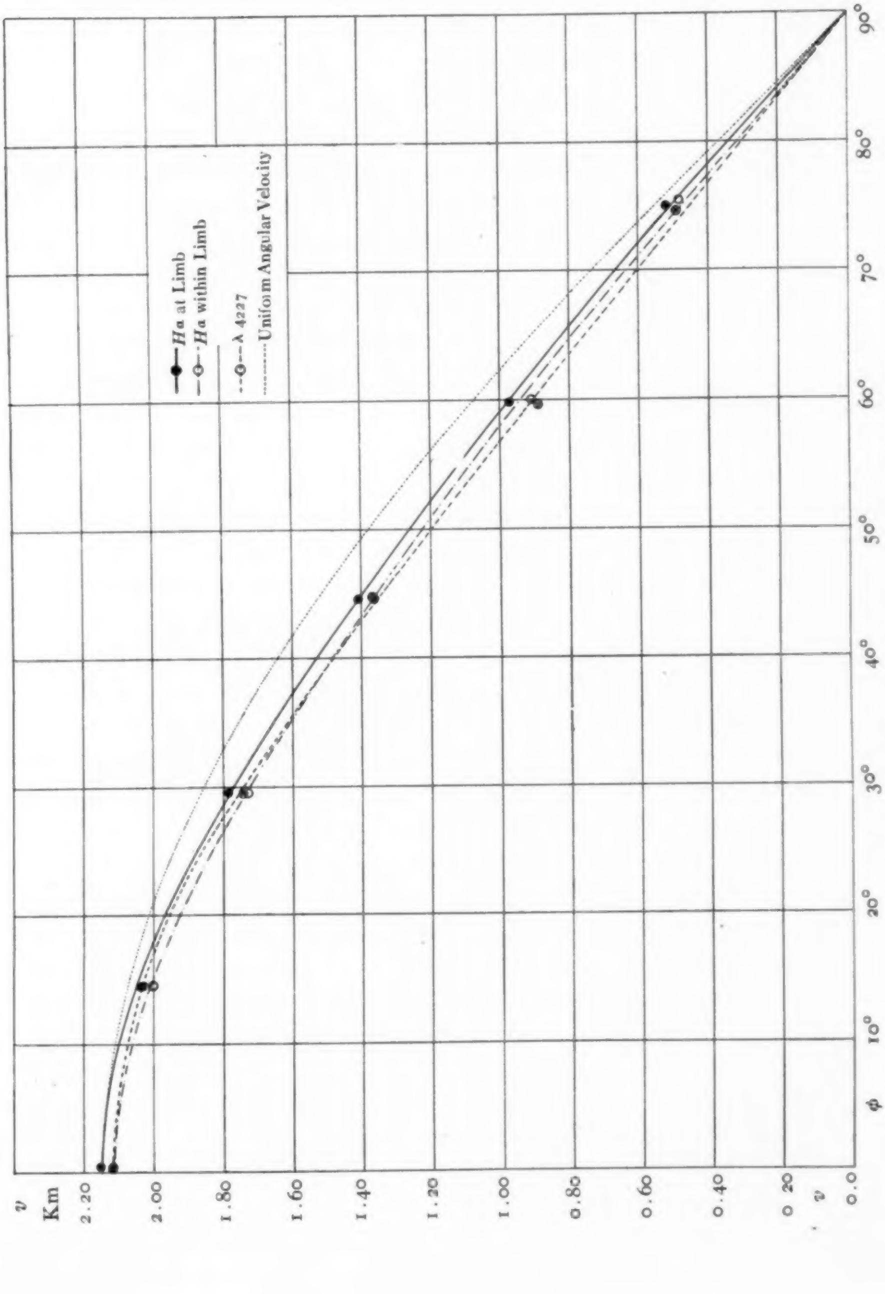



FIG. 2.—Radial Velocity Curves for *Hα* and λ 4227

agreeing closely with those of 1906–1907 between latitudes 0° and 50°. Above 50° they give larger values, the difference in linear velocity reaching at a maximum 0.036 km.

2. The general agreement of the results, and the excellent accord with Dunér's values, are opposed to the existence of a variation in the rotation rate between 1906 and 1908. If any such variation exists it is confined to the higher latitudes, and does not appear in the zones of greatest spot activity. The results are also opposed to a three-year period of variation, such as was obtained by Halm from a comparison of his values with those of Dunér.

3. The observations of 1908 confirm those of 1906–1907 in showing that different lines give different velocities. Lines of lanthanum and cyanogen give low velocities; certain lines of manganese and iron give high velocities. The investigation of two "enhanced" lines indicates a tendency toward low values for lines of this type. In one of the cases this effect is very marked. Lines considerably strengthened at the sun's limb give high values in general.

4. When lines give systematically large or small values for the rotational velocity the differences from the mean become greater toward higher latitudes.

5. The results given by the 1908 observations are satisfied within the limits of accidental error by the equation given by Faye for the motion of the sun-spots observed by Carrington. The fact that the observations of Dunér, Halm, and myself are all satisfied by this equation indicates that this represents the law of rotation of the sun's reversing layer to within at least 10° of the pole.

6. Comparison of the probable errors for the two series of observations indicates a substantial gain in accuracy of measurement for the 1908 series over that of 1906–1907.

7. The motion of the reversing layer in the vicinity of solar vortices may be seriously influenced by the motion of the vortices, and the rotation velocities obtained from such regions are subject to large systematic errors. It is important in taking observations for rotation to avoid all such disturbed areas.

8. A special study of the calcium line $\lambda$ 4227 shows that the rotational velocity derived from this line is higher than that for the general reversing layer, the difference at the equator amounting to 0°.3. Also the decrease of velocity with increasing latitude is much

less marked than for the reversing layer. At 75° of latitude the angular velocity of $\lambda$ 4227 is 1°5 greater than for the reversing layer.

9. A special study of the $a$ line of hydrogen shows that the rotational velocities which it gives depend upon the distance from the sun's limb. Results obtained from $H\alpha$ at the limb of the sun are considerably larger than those for the reversing layer and show a comparatively small decrease in the value of the angular velocity toward the pole. At the equator the difference from the reversing layer amounts to 0°6, and at 75° of latitude to 3°0.

10. At a distance averaging 35″ inside the sun's limb the results obtained from $H\alpha$ are considerably smaller than the corresponding values at the limb, although still much larger than for the reversing layer. They also average somewhat larger than for $\lambda$ 4227.

11. The sudden increase in angular velocity at latitude 75° may perhaps be a genuine effect similar to that found among the lines of the reversing layer which give systematic deviations.

12. The large rotational values given by $\lambda$ 4227 and $H\alpha$, as well as the differences found for $H\alpha$ at the limb and inside the limb, may all be accounted for upon the basis of differences of level in the solar atmosphere.

It may be of interest, acting on a suggestion from Professor Kapteyn, to call attention to the fact that the difference in the values of the rotation obtained from different spots on the planet *Jupiter* may perhaps be explained on a basis similar to that given for different elements in the sun. A difference of about 6 minutes has been found in the rotation period given by different spots upon the surface of *Jupiter*.[1] The ratio of this difference to the longer rotation period is of much the same order as that obtained by comparing $\lambda$ 4227 with the reversing layer, or $H\alpha$ with $\lambda$ 4227. It seems reasonable to conclude, therefore, that the different velocities found in *Jupiter's* atmosphere may be wholly accounted for upon the basis of a difference of level for the various spots observed.

I wish to express my appreciation to Miss Lasby for her careful and accurate measurement of the majority of the plates included in this discussion.

MOUNT WILSON SOLAR OBSERVATORY
November 1908

[1] Stanley Williams, *Monthly Notices*, **56**, 143–151, 1896.

# STUDIES IN SENSITOMETRY. III

## ON THE EVALUATION OF THE RECIPROCITY LAW, BASIC FOG AND PRELIMINARY EXPOSURE

By R. JAMES WALLACE and HARVEY B. LEMON

A cursory examination of past photographic literature is sufficient to show the recurrence of similar ideas and methods which have either been discarded as fallacious, or, because of lack of definite scientific investigation, have never advanced beyond a somewhat nebulous stage of development. The systematic and unbiased investigation of these ideas is of the utmost importance, not only in the upbuilding of a firm foundation upon which may be reared the structure of photographic practice, but such investigation is also of major importance to every scientist who records his results by photography.

The work detailed in the present paper was undertaken in the hope of obtaining definite measurable results and shedding light upon somewhat disputed questions.

Primarily, it was concerned merely with the effect of a supplementary "fogging" exposure upon a plate as influencing the recording power of the sensitive film, but, as the work progressed, it was found necessary to investigate also the influence of varying amounts of "fog" upon recorded densities, and likewise the so-called "reciprocity law."

It is usual in all photographic density measurements to subtract from each reading the total amount due to basic "fog," i. e., the sum of the absorption of the glass, the gelatine, and the general reduction products in the film. As a usual thing this value is comparatively small, amounting in a good, clean-working plate to about 0.05 (Hurter and Driffield), which is practically negligible except on records of small total luminosity. From causes either inherent in the film—induced by subject or apparatus, or intentionally added (as in the case of a preliminary exposure)—this value may increase in an alarming manner and rise as high as 0.45 or 0.5 H. and D. This is equivalent to 20 per cent. of the highest allowable printing

PLATE XIII

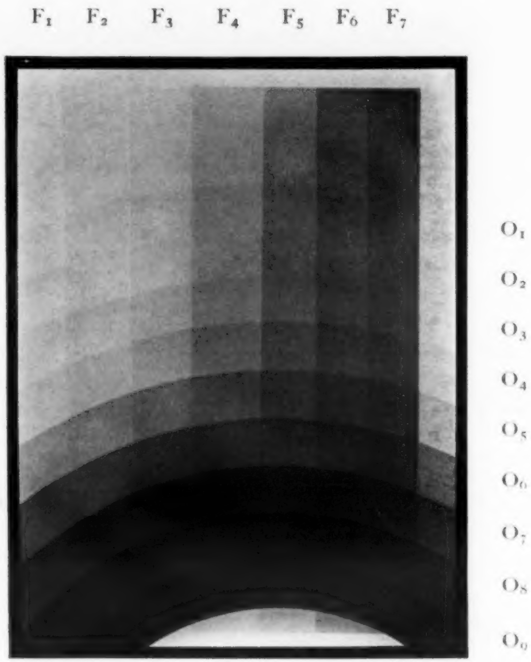



Fig. A

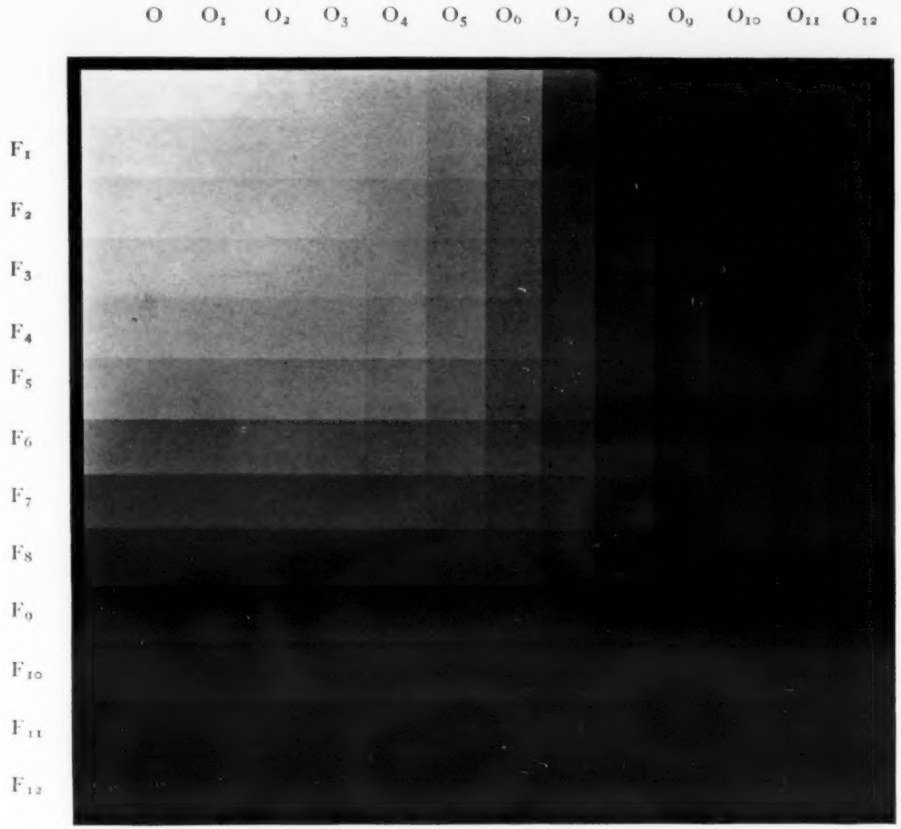



Fig. B

density upon the plate, for when this fog value is subtracted from the various densities, the records are not only in erorr, but result in anomaly.

As an example of such, the following experiment is detailed.

A Seed "G. E. 27" plate was impressed with a series of preliminary exposures in strips $A_1$, $A_2$ . . . . $A_8$ running parallel to the longer edge. The plate was then exposed in the sector-disk machine and received a second series of exposures $B_1$, $B_2$ . . . . $B_8$, and was developed. Measurements of the plate gave the following density values:

TABLE I

| No. | Original (Minus Fog) | Original + $F_3$ | Original + $F_6$ |
|---|---|---|---|
| Fog* | 0.0105 | 0.2418 | 0.5278 |
| 1 | .0105 | .0216 | .0094 |
| 2 | .0105 | .0386 | .0286 |
| 3 | .0461 | .0700 | .0486 |
| 4 | .1841 | .1662 | .1038 |
| 5 | .4327 | .3472 | .1792 |
| 6 | .7603 | .6110 | .4082 |
| 7 | 1.1669 | 1.0032 | .6988 |
| 8 | 1.6443 | 1.4504 | 1.1328 |
| 9 | 2.2223 | 1.9130 | 1.6634 |

* The value of the "fog" on subscripts 3 and 6 equals the value of the preliminary exposure plus the basic "fog" of the original.

If now these measurements be plotted in the usual manner against log exposure, it will readily be seen (Fig. 1) that all of the curves will cross the original curve at a low density-value, so that in the region of higher density these results are utterly false representations of the plate values; for these, while still in the straight portion of the characteristic curve, are shown as of lower density when the incident light-value is increased. The utter falsity of this result may be appreciated by glancing at Fig. A, Plate XIII, which is a reproduction of a similar negative.

Observation of Fig. A will at once impress the fact that as the increasing supplementary fog value $F_n$ falls upon successively increasing original densities ($O_n$), the additive effect becomes less and less apparent until it is finally lost; this *extinction point* (so to name it) varying with the relative values of $F$ and $O$. It is therefore evident that the subtraction of an equal value for this fog ($F$) from densities

$O_n$ cannot be considered. To determine this variation was therefore obviously the first consideration.

### "FOG" VALUE WITH INCREASING DENSITIES

Upon a Seed "27" plate was made a series of exposures to the constant acetylene light, the exposures being so timed that each strip received an exposure increasing by 1.27 times that immediately preceding it, and then developed. When dry the plate was measured in the spectrophotometer at several points along the *length* of the

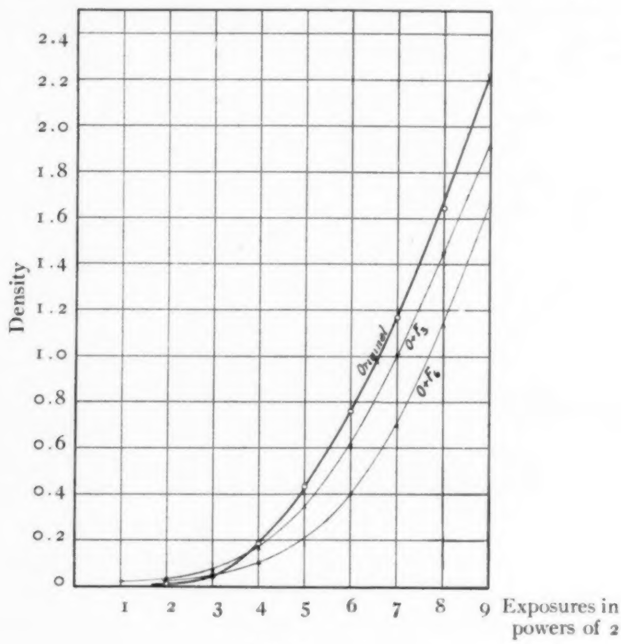



FIG. 1

strips in order to determine the evenness of the deposit.[1] This plate was then used as a transparency behind which were exposed a number of other plates for fog value determination.

This transparency plate was set up at a distance of 2.0 meters from the constant acetylene light, and the plate behind and in contact with it received an exposure of 200 seconds. Upon completion of the exposure the plate was rotated 180° and exposed again for

[1] Owing to the coating inequality of the ordinary commercial plate, it was found necessary to make careful selection before one sufficiently even was found.

another 200 seconds and then developed. A large number of such plates was made in the course of the investigation.

Measured densities were then plotted as ordinates against log exposure as abscissae, and as the actinic value of the acetylene burner at a distance of 2.0 meters=0.15 candle, then

$$\log E = \log I + \log T,$$

where log $T$ is the value obtained by measurement of the strips on the transparency plate.[1] These curves are shown in Fig. 2.

Referring to Plate XIII and calling one series of exposures the original, its successive strips may be designated $O, O_1, O_2, \ldots . O_{20}$, while the other exposures may be termed "fogging strips" and designated as $F_1, F_2, F_3, \ldots . F_{20}$; It will of course result that the graphic illustration of $O, O_1, O_2, \ldots . O_{20}$ (which has been protected from any additive exposure) will simply represent the characteristic curve of the plate, while $F_1$ $(O, O_1, O_2, O_3, \ldots . O_{20})$ illustrates the change occurring when definite amount of "fog" $(F_1)$ is added to the original series of exposures $O, O_1, O_2, \ldots . O_{20}$ (Fig. 2).

In sensitometric and general scientific work where it very often occurs that heavily fogged plates have to be measured, the value of the curves should be obvious: particularly in the measurement of relative brightness in such instances as solar disk phenomena, selective spectral intensity, etc.

In use, one knowing the value of the "fog" under consideration, may at a glance obtain the "minus value" of the same for densities of varying amount. It will be noted that fog increase values appear to become asymptotic to the straight portion of the characteristic curve.

### GAIN FROM PRELIMINARY EXPOSURE

To determine the value of preliminary exposure with reference to the photographic visibility of light of feeble intensity, the following method was adopted. Exposure was made to the constant acetylene light through a transparency graduated in strips (whose densities advance in arithmetical progression) for a definite length of time, and at a definite distance. The plate was next turned through 180° and exposed behind the rotating sector disk, the apertures of

[1] A series of differently timed crossed exposures made directly to the acetylene light without the interposition of the "transparency" gave precisely similar results.

which advance geometrically. Both exposures were so timed that the developable impression ceased to be visible before exhausting the

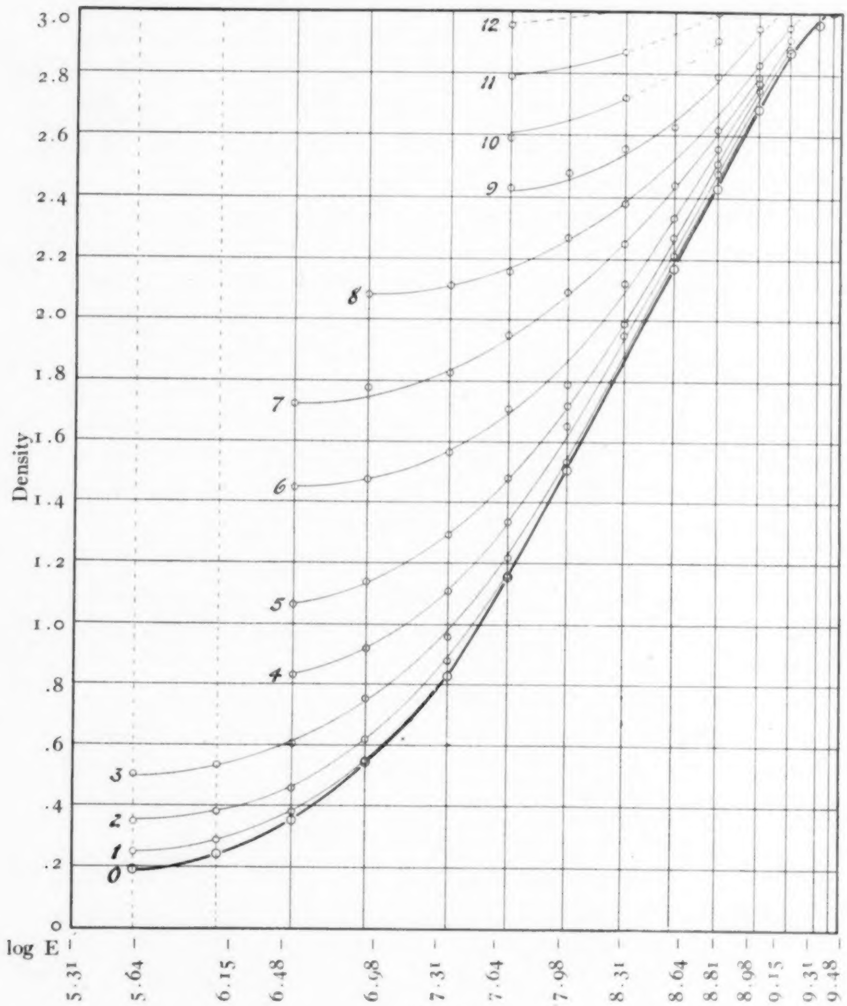



FIG. 2

| | | | |
|---|---|---|---|
| $O$. | Original (unfogged) | $F_6$. | Density = 1.4006 |
| $F_1$. | Density = 0.2154 | $F_7$. | " = 1.7128 |
| $F_2$. | " = .3072 | $F_8$. | " = 2.0060 |
| $F_3$. | " = .4872 | $F_9$. | " = 2.3130 |
| $F_4$. | " = .7482 | $F_{10}$. | " = 2.5340 |
| $F_5$. | " = .9970 | $F_{11}$. | " = 2.6866 |
| | | $F_{12}$. | " = 2.9144 |

number of strips. The plate was then developed as usual under constant conditions.

A very large number of plates (Seed "27") were thus made with the light at different distances and behind variously graduated transparencies, but the results in every case simply verified one another.

Fig. A, Plate XIII, is a reproduction of such a plate as has just been described, and even with the unavoidable loss due to the half-tone process it will be seen that there are arcs showing upon some of the "fogged" sections which are totally invisible upon the original unfogged region. This gain is best noticeable at $F_4$ and the gradual disappearance of the effect of the additive "fog" against increasing density of the original is also clearly shown.

All of the plates thus exposed were assembled and selection for measurement was made of that one which showed the maximum effect of increase. The measurement of such small differences requires the utmost care and is very trying upon the eyes; ample precautions were taken relative to screening off extraneous light.

The density measures of this plate are shown in Table II, and the plotted values in Fig. 3.

TABLE II

| No. | Original* (Unfogged) | Original† + Prelim. Exposure |
|---|---|---|
| 1 | | 0.4938 |
| 2 | | .5232 |
| 3 | 0.1588 | .5590 |
| 4 | .2818 | .6512 |
| 5 | .5324 | .7646 |
| 6 | .8306 | .9970 |
| 7 | 1.2574 | 1.3860 |
| 8 | 1.7128 | 1.8038 |
| 9 | 2.2108 | 2.2108 |

* These values represent the actual density, the value of the density due to glass and gelatine having been subtracted, as is usual in sensitometric measures.

† Density of preliminary fog = 0.4512.

It was at first intended to plot the densities of this plate against actual candle power, but further consideration of the method showed the infeasibility of such a plan, for while it presumably would be true for a certain emulsion evenly coated, yet commercial production does not allow of such a degree of accuracy, and any change in the

speed or constitution of the emulsion would necessitate an entirely new determination. We considered it decidedly better, therefore, to plot these densities against exposure ratios in powers of 2 as being the more practical, and also because this method presents uniformity with earlier papers published by one of us.

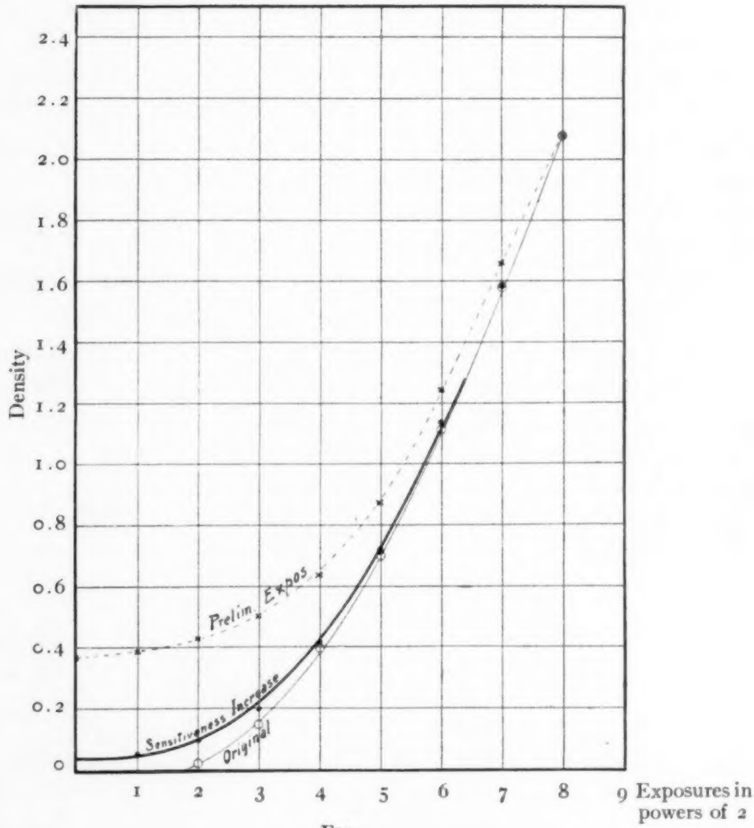



FIG. 3

Considering now these curves shown in Fig. 3, the continuous line represents the original (unfogged), while the dotted line represents the same original exposure but as influenced by an amount of preliminary exposure giving a density value of 0.3218. It is of course understood that this density (0.3218) would appear in practice entirely covering the plate as a "basic fog" and would be representative of the highest transparency thereon. Having a definite

measure of this additive fog, we can, by reference to the curves shown in Fig. 2, construct another curve which will represent the true increase or gain in sensitiveness conferred by the preliminary exposure. This curve is shown by the heavy continuous line in Fig. 3.

Just how much has been gained by the method is plainly shown graphically, but is difficult to present in figures. One naturally expects to be able to state in percentages the amount of any change, but in this instance such a method would convey very misleading information; for, while with the most careful examination No. 3 was the first exposure to show any trace of development action on the original (unfogged) scale, yet Nos. 1 and 2 showed perfectly measurable differences on the strip preliminarily fogged. The gain could thus be only represented in percentage by "infinity;" which is ridiculous. We have considered it better therefore to make our deductions from sector exposure No. 3, which gives a density difference of 0.08 (H. and D.). This value corresponds to about 0.2 magnitude on stars near the limit of photographic action, i. e., of a density so low that their impress is but barely discernible on the negative. This gain, as will be seen both from the curve and from Plate XIII (B), is a quantity decreasing with the density: for example, for neighboring stars giving a density value of 0.4 the gain is reduced to 0.08 magnitude.

From these results it therefore appears to us that there is no *practical* advantage in preliminary (or supplementary)[1] exposure in so far as concerns astronomical (stellar) photography. It is, however, perfectly comprehensible that there may be special occasions (more particularly in long spectroscopic exposures) where the exposures are measured by hours or days, when the slight gain would be well worthy of consideration; *ordinarily* the "gain" is a negligible quantity which is in many lines of work offset by the change in gradation curve.[2] The great majority of workers will naturally hesitate to purposely "fog" their plates for so slight a gain.

[1] Plates made with supplementary exposure resulted in values similar to those preliminarily exposed.

[2] This change is shown by the uniformly lower $\gamma$ of the "fogged" strip, i. e., by a smaller angle between the straight portion and the log $E$ axis.

RECIPROCITY LAW

The well-known Bunsen-Roscoe "reciprocity law" states that the product $It=E$, where $I$ is the intensity, $t$ the time, and $E$ the exposure. Abney was the first to point out that this law did not hold for photographic plates, and further work has since been undertaken upon the subject by English, Schwarzschild, Mees and Sheppard, and others, from which it results that the formula $It^P=E$ is more in agreement with truth when $P$ is slightly less than unity.

As the matter is of considerable importance from an astronomical standpoint, it was decided to verify these foregoing values upon the plates most generally in use[1] in astronomical and general scientific work, viz., Seed "27" and Cramer "Instantaneous Isochromatic."

A large number of exposures were made upon these respective plates behind the graduated transparency before mentioned, at different distances from the constant acetylene light, and (by means of shutters) for different lengths of time, i. e., different values of $P$. From the formula $It^P=E$ we obtain:

$$I_1 t_1^P = I_2 t_2^P ,$$

whence

$$P = \frac{\log \frac{I_1}{I_2}}{\log \frac{t_2}{t_1}} ,$$

and

$$t_2 = \text{antilog}\left(\frac{\log I_1 - \log I_2 + P \log t_1}{P}\right) .$$

In actual practice six separate exposures were made on each plate ("27" and "iso") in the following manner. One portion of the plate was exposed behind the transparency for 40 seconds with the constant light at a distance of 2.0 meters. This "constant" exposure may be represented by $m$. The light was then removed to a distance of 12.0 meters ($=\frac{1}{36}$ the original intensity) and 4 more exposures were given on the remaining portions. As each exposure time was completed, the movable slides protected that portion from further light-action, which, for example, were in some cases

[1] In the United States.

| | | | |
|---|---|---|---|
| 24 minutes ($=m$) | | | $=P=1.0$ |
| 35 " | 4 seconds | | $=P=0.905$ |
| 53 " | 48 " | | $=P=0.816$ |
| 1 hour 26 " | 16 " | | $=P=0.734$ |
| 2 " 27 " | 52 " | | $=P=0.663$ |

although varying values for $P$ were given on different plates.

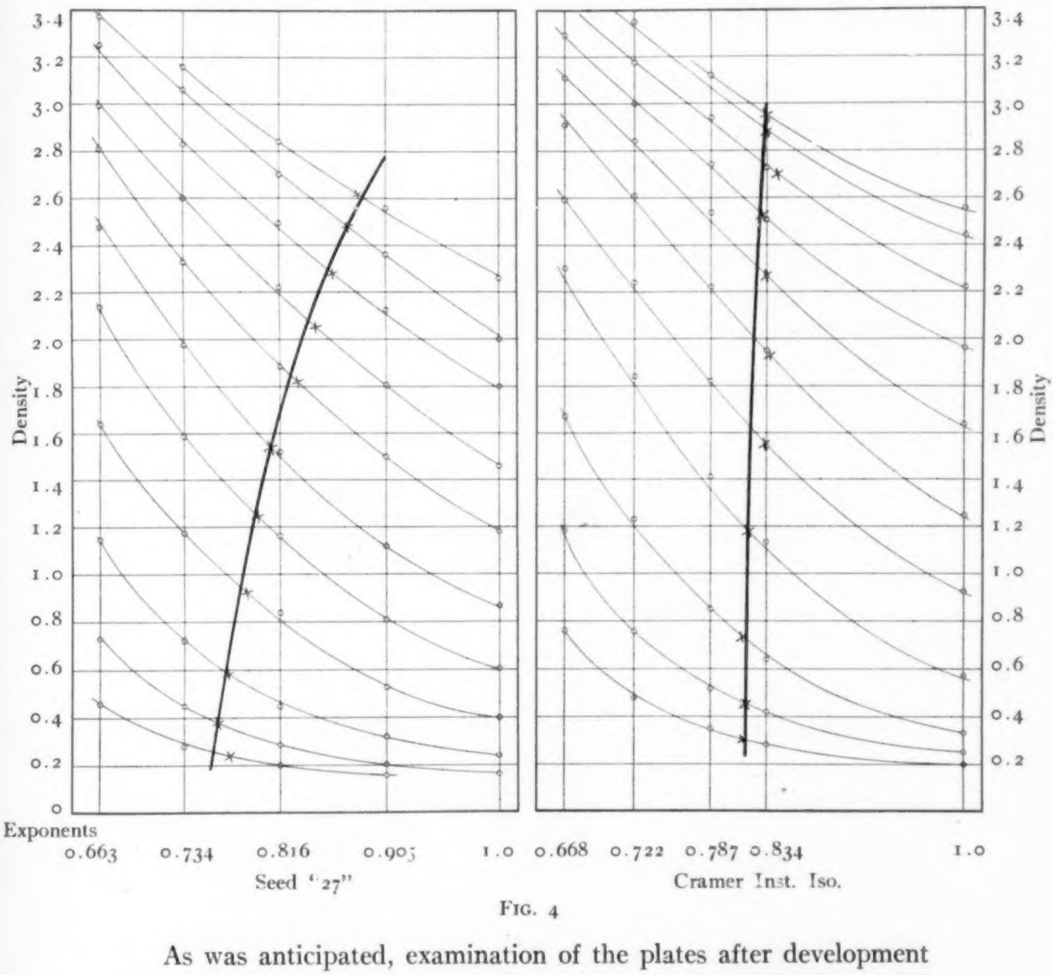



FIG. 4

As was anticipated, examination of the plates after development showed that in every case, with the "27," the exponential value varied with the density. *With the "iso" plate this variance does not obtain,* but remains practically constant even beyond the limit of full printing

density. Plotting the measures made from these two types of plate against the true log $E$, the crossing of the various curves over $m$ is plainly shown.

If, however, these curves be plotted with density against exponents, we obtain a series of smooth curves which present great interest, for upon each unit we may indicate the density value of $m$, and, connecting the points, obtain a result which gives directly the exponential value required to reproduce a density of any chosen magnitude. Thus it will be noted (Fig. 4) that with a Seed "27" plate for a density value of 2.6 the exponent value required is 0.88, while for $D=0.4$, $P=0.76$.

In astronomical photography, assuming that for certain stars of $n$th magnitude an exposure of 60 minutes is necessary in order to obtain upon the film a density of 0.4, then the exposure time necessary to obtain images of comparable density from stars 1.0 magnitude fainter would be found as follows:

Let

$$I_1=1$$

$$I_2=\frac{1}{2.5}$$

$$P=0.9$$

$$t_1=60 \text{ minutes}$$

$$t_2=x$$

then

$$\frac{0+0.3979+0.9\times 1.7782}{0.9}=2.22$$

and

$$\log^{-1} 2.22=166 \text{ minutes}=t_2 .$$

For stars of higher density the time is proportionately shorter.

Regarding the Cramer "inst. iso." plate curves (Fig. 4), it will be seen that when plotted in similar manner the exponential curve is practically a straight line with its mean value at 0.83, which is in practical agreement with Schwarzschild, who found when using Schleussner plates $P=0.86$. It is evident, however, that the value for $P$ will vary with plates of identical "brand" if the emulsion be not coated in one operation at the one time. It is also probable that the $P$ value may vary with the wave-length of the incident light.

YERKES OBSERVATORY
November 30, 1908

# THE ARC SPECTRUM OF IRON $\lambda$ 6855 to $\lambda$ 7412

BY E. J. EVANS

Some time ago Professor Fowler suggested to me the investigation and tabulation of the wave-lengths of the titanium lines in the red end of the spectrum.

Since the spectra first studied were prismatic, it was necessary to know the wave-lengths of certain lines in the above region, which were to be employed as standards to calculate the constants $\lambda_o$, $c$, and $s_o$, in the Cornu-Hartmann formula, $\lambda = \lambda_o + \frac{c}{s - s_o}$.

With this end in view the iron spectrum was investigated in the same region. The instrument employed was a spectrograph of the Littrow pattern having one prism of angle 60° ($\mu_D = 1.6467$), and an object glass of 12 ft. focal length. The photographs were taken with Messrs. Wratten's panchromatic plates. The plates were then measured on a stage-micrometer reading to 0.001 mm by estimation, and the reduction to wave-lengths was made by means of the above formula.

Plates of the iron spectrum taken with long exposure showed several lines of greater wave-length than 6855.419, the last iron line tabulated by Rowland. By extrapolation approximate wave-lengths were obtained for these lines. It was immediately noticeable that the wave-lengths fell very near to those of lines in the solar spectrum, which were then assumed to be due to iron vapor. The solar lines 7207.715, 7038.50, and 6855.419 were now taken as standards for the calculation of the constants for interpolation, and the wave-lengths of the other lines determined. The wave-lengths thus calculated were coincident within about 0.04 of a tenth-meter with the solar lines.

It afterward became possible to investigate the iron spectrum with the aid of a Rowland concave grating of 10 ft. radius, and about 14,500 lines to the inch. The temporary mounting of the grating for this work did not give spectra which were strictly normal, but a linear formula could be used for interpolating between standards not far

apart. With this instrument a first-order iron spectrum of one hour's exposure was taken, and overlapping it a second-order spectrum of iron. In this way the wave-lengths of some of the lines were

TABLE OF WAVE-LENGTHS OF IRON LINES FROM 6855 TO 7412

| Wave-Length | Int. | Nearest Solar Line (Rowland) | Origin (R) | Int. (R) | Remarks |
|---|---|---|---|---|---|
| 6855.419 | 10 | 6855.419 | *Fe* | 3 | Taken as standard |
| 57.50 | 1 | 57.515 | | 0 | |
| 58.42 | 5 | 58.415 | | 2 | |
| 62.12 | 0 | { 62.014 | | 0000 | |
| | | { 62.210 | | 0 | |
| 62.76 | 1–2 | 62.76 | | 1 | |
| 76.23 | 0 | 76.255 | | 000N | |
| 80.90 | 0–1 | 80.887 | | 00 | |
| 81.82 | 1 | 81.720 | | 00 | |
| 86.05 | 4–5 | 86.000 | A(O) | 11 | |
| 98.56 | 1–2 | 98.556 | | 00 | |
| 6903.10 | 1–2 | 6903.120 | | 00 | |
| 11.83 | 0 | 11.790 | A, – | 00N | |
| 15.43 | 0 | | | | |
| 16.96 | 7 | 16.948 | | 2 | |
| 30.89 | 0 | 30.890 | | 00N | |
| 33.33 | 0 | 33.305 | | 0N | |
| 33.89 | 2 | 33.887 | | 2 | |
| 45.46 | 10 | 45.477 | | 4 | |
| 47.72 | 1 | 47.702 | A(O) | 0 | |
| 51.54 | 5 | 51.518 | | 1 | |
| 60.60 | 0 | 60.590 | | 00 | |
| 72.17 | 0 | 72.205 | | 00 | |
| 75.80 | 1 | 75.706 | | 0 | |
| 76.57 | 0 | 76.535 | | 000N | |
| 77.17 | 0 | 77.198 | | 00 | |
| 77.72 | 1 | 77.715 | A(WV) | 3 | |
| 79.12 | 10 | 79.120 | | 2 | Taken as standard |
| 88.79 | 2 | 88.805 | | 0 | |
| 7000.16 | 5 | 7000.155 | | 1 | |
| 00.91 | 1 | 00.880 | | 00 | |
| 08.26 | 2 | 08.225 | | 0 | |
| 10.65 | 1 | 10.618 | | 000N | |
| 11.67 | 1–2 | 11.590 | A(WV) | 2 | |
| 16.32 | 2 | 16.330 | – A(WV) | 1 | |
| 16.67 | 6 | 16.675 | –,A(WV) | 3 | |
| 23.25 | 5 | 23.230 | | 2 | |
| 24.32 | 1 | 24.340 | | 0 | |
| 24.90 | 2 | 24.913 | | 1 | |
| 38.50 | 5 | 38.500 | | 1 | |

obtained, and their coincidence with the previously mentioned solar lines confirmed.

The photographic plates were measured on the stage-micrometer, and in the final reductions the solar wave-lengths as obtained from

Rowland's tables were employed. As far as could be determined the specimen of iron employed was almost pure.

The grating photograph brought out about 90 lines in the spectrum below 6855.419. Some of these were too faint to be accurately measured, and have therefore been omitted in the appended table.

The wave-lengths of the nearest solar lines, with their intensities, have been included in the table. The values taken are those given by Rowland, and the origins of the lines (if any) as given by him have also been included.

TABLE OF WAVE-LENGTHS OF IRON LINES FROM 6855 TO 7412

| Wave-Length | Int. | Nearest Solar Line (Rowland) | Origin (R) | Int. (R) | Remarks |
|---|---|---|---|---|---|
| 7039.08 | 0 | 7039.040 | | 0 | There is a *Ti* line at 7039.04 |
| 68.67 | 5 | 68.685 | | 2 | |
| 72.13 | 0 | 72.130 | | 0N | |
| 83.68 | 0 | 83.680 | | 00N | |
| 87.03 | 1 | 87.008 | | 00 | |
| 90.66 | 5 | 90.660 | | 2 | |
| 95.68 | 1 | 95.690 | | 0 | |
| 7107.76 | 0 | 7107.740 | | 0 | |
| 12.46 | 1 | 12.450 | | 0 | |
| 31.21 | 6 | 31.204 | | 3 | |
| 33.29 | 2-3 | 33.263 | | 1 | |
| 45.64 | 1 | 45.615 | | 1 | |
| 56.00 | 0 | 55.945 | | 0N | |
| 64.76 | 6 | 64.725 | –, A | 2 | |
| 76.25 | 1 | 76.225 | A, – | 1 | |
| 77.21 | 0 | 77.170 | A | 0 | |
| 81.54 | 2-3 | 81.500 | A | 0 | |
| 82.27 | 0 | 82.260 | A? | 00 | |
| 87.65 | 10 | 87.645 | –, A | 5N | |
| 7207.715 | 8 | 7207.715 | | 1 | Taken as standard |
| 20.08 | 1 | 19.987 | | 0 | The wave-lengths of these lines were obtained by extrapolation and the lines 7390 and 7412 may be incorrect to about 0.3 tenth-meter |
| 21.57 | 0 | 21.492 | | 0 | |
| 24.02 | 2 | 23.930 | A | 3 | |
| 40.39 | 1 | | | | |
| 89.21 | 0 | | | | |
| 93.50 | 1 | | | | |
| 7390.14 | 2 | | | | |
| 7412.05 | 2 | | | | |

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY
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# THE ARC SPECTRUM OF TITANIUM FROM λ 5866 TO λ 7364

By E. J. Evans

The arc spectrum of titanium in the region more refrangible than λ 5899 has already been very fully investigated by Hasselberg,[1] but although Rowland and Thalén have dealt with the less refrangible part of the spectrum the list of lines in this region has hitherto been incomplete. In view of the importance of titanium in connection with the spectra of sun-spots, the present investigation was undertaken to supply the missing data, especially in the region 6800 to 7360.

The light obtained from an arc between two carbon or iron poles, the positive of which had been charged with titanium oxide, was first analyzed by the Littrow spectroscope described in the previous paper.[2] The photographs were measured on the stage-micrometer, and the wave-lengths calculated in the usual way.

The iron-pole spectra of titanium gave a very convenient method of obtaining the wave-lengths of the titanium lines without taking a comparison spectrum, as some of the iron lines were photographed simultaneously with those of titanium, and there was no danger of relative displacements. The wave-lengths of iron lines as determined in the previous investigation were used for the calculation of the constants of the Cornu-Hartmann formula, from which the wave-lengths of the titanium lines could be ascertained.

The dispersion given by the prism in the extreme red was comparatively small, and therefore the titanium spectrum in this region was also studied with the previously described Rowland concave grating when it became available. A first-order titanium spectrum with an overlapping second-order iron spectrum was photographed, and the wave-lengths of a few titanium lines calculated. The strong lines were identified with solar lines tabulated by Rowland, and the solar wave-lengths were adopted in making the linear interpolations

[1] *Köngl. Svenska Vetenskaps-Akadem. Handl.*, Bd. **28**, No. 1 (1895).

[2] "The Arc Spectrum of Iron from λ 6855 to λ 7412," p. 157.

TABLE OF TITANIUM LINES (λ 5866 to 7364)

| Wave-Length | Int. | Nearest Solar Line (Rowland) | Origin (R) | Int. (R) | Remarks |
|---|---|---|---|---|---|
| 5866.675 | 6 | 5866.675 | *Ti* | 3 | Used as standard for Cornu-Hartmann formula |
| 77.99 | 1 | 78.015 | | 0 | |
| 80.48 | 2 | 80.490 | | 00 | |
| 99.56 | 5–6 | 99.518 | *Ti* | 1 | |
| 5903.56 | 1–2 | 5903.555 | | 00 | |
| 18.79 | 3–4 | 18.773 | *Ti* | 0 | |
| 22.35 | 4–5 | 22.334 | *Ti* | 0 | |
| 38.03 | 2–3 | 38.035 | | 000 | |
| 41.97 | 4 | 41.985 | *Ti* | 00 | |
| 53.386 | 5 | 53.386 | *Ti* | 1 | Taken as standard line |
| 66.08 | 5 | 66.055 | *Ti*, A? | 2 | |
| 78.79 | 5 | 78.768 | *Ti* | 1 | |
| 96.11 | 1 | 96.164 | | 000N | |
| 99.13 | 0–1 | 99.115 | | 0000 | |
| 99.90 | 2–3 | 99.920 | *Ti*,A(WV) | 0 | |
| 6002.92 | 0 | 6002.97 | | 0000N | Probably a fluting line |
| 18.78 | 0–1 | 18.76 | | 0000 | Probably a fluting line |
| 64.853 | 4 | 64.853 | *Ti* | 00 | Taken as standard |
| 85.48 | 5 | 85.47 | *Ti*, *Fe* | 2 | |
| 91.40 | 5 | 91.395 | | 0 | |
| 93.00 | 2 | 93.03 | | 000 | |
| 98.92 | 3 | 98.87 | | 00 | |
| 6121.24 | 2 | 6121.215 | | 000 | |
| 26.435 | 5 | 26.435 | *Ti* | 1 | Taken as standard |
| 38.64 | 0–1 | 38.725 | | 00N | |
| 46.48 | 2 | 46.445 | | 000 | |
| 50.02 | 1–2 | 49.95 | | 0000N | |
| 86.65 | 2 | | | | |
| 6215.630 | 3–4 | 6215.630 | *Ti* | 000 | Taken as standard |
| 20.87 | 2 | | | | |
| 21.75 | 1–2 | 21.552 | *Fe*– | 00Nd? | |
| 58.60 | 9 | { 58.322<br>58.927 | *Ti*<br>*Ti* | 2<br>3 | 6258.29 } were wave-lengths de-<br>6258.96 } termined by grating |
| 61.316 | 6 | 61.316 | *Ti* | 1 | Taken as standard |
| 6303.97 | 3 | 6303.985 | | 000N | Given by Hale as strong *Ti* fluting line |
| 12.47 | 3 | 12.456 | | 00N | Given by Hale as strong *Ti* fluting line |
| 18.26 | 2 | 18.239 | *Fe* | 6 | This line comes out on carbon poles |
| 36.329 | 3 | 36.329 | *Ti* | 000N | Taken as standard |
| 66.59 | 4 | 66.564 | *Ti* | 000 | |
| 6497.92 | 1–2 | 6497.840 | A(WV) | 00Nd? | |
| 6505.72 | 0–1 | 6505.71 | | 0000N | Possibly a fluting line |
| 08.37 | 1–2 | 08.380 | | 0000 | |
| 36.74 | 0–1 | | | | Possibly a fluting line |
| 6546.479 | 4 | 46.479 | *Ti*, *Fe* | 6 | Taken as standard |
| 48.57 | 1 | | | | |
| 54.47 | 5 | 54.470 | *Ti* | 0 | |
| 56.31 | 5 | 56.308 | *Ti*,A(WV) | 1 | |
| 65.84 | 2–3 | 65.783 | A? | 000 | |
| 75.39 | 1–2 | | | | |
| 99.353 | 4 | 99.353 | *Ti* | 00 | Taken as standard |
| (6651.55) | 1 | | | | Fluting head. Hale gives 6651.56 |
| 66.82 | 1–2 | 6666.78 | | 000N | |

TABLE OF TITANIUM LINES—*Continued*

| Wave-Length | Int. | Nearest Solar Line (Rowland) | Origin (R) | Int. (R) | Remarks |
|---|---|---|---|---|---|
| 6668.04 | 1–2 | | | | Hale 6668.02. Spot 6668.03 |
| 68.68 | 1–2 | 6668.640 | | 000N | |
| 77.48 | 0–1 | | | | |
| 6719.90 | 0–1 | 6719.880 | | 000N | Hale gives 6719.86 as strong *Ti* fluting line |
| 43.380 | 5 | 43.381 | *Ti* | 1 | |
| 64.18 | 1 | | | | |
| 6844.93 | 1 | 6844.939 | | 0000 | Possibly fluting lines |
| 50.48 | 1–2 | | | | |
| 54.55 | 1 | 54.590 | | 000 | |
| 61.770 | 3–4 | 61.770 | *Ti?* | 000 | |
| 74.24 | 2 | | | | May be a fluting line |
| 6911.43 | 2–3 | | | | Strongest near the pole. *Ti?* |
| 13.51 | 2 | 6913.448 | A(O) | 11 | According to Rowland 6913.284 is *Ti?* |
| 26.41 | 2 | 26.363 | | 0 | |
| 33.42 | 1–2 | 33.469 | | 0000 | |
| 39.13 | 3–4 | 39.050 | | 000 | Strongest near the pole. *Ti?* |
| 43.96 | 2 | 43.904 | | 0000 | |
| 96.91 | 2–3 | 96.93 | | 0000 | |
| 7004.87 | 1–2 | | | | |
| 06.95 | 0–1 | 7006.915 | A | 000 | Fluting line? |
| 08.64 | 1–2 | | | | |
| 11.27 | 2 | 11.230 | A | 000N | |
| 36.09 | 2–3 | 36.129 | | 0000 | |
| 39.05 | 4 | 39.040 | | 0 | |
| 39.59 | 1 | 39.560 | A | 0N | May be fluting line |
| 50.92 | 1–2 | 50.783 | A? | 000 | |
| (54.56) | 2 | | | | Strong flut. head { Hale 7054.65, Spot 7054.60 |
| 65.41 | 1 | 65.49 | | 0000 | May be fluting line |
| 69.35 | 2–3 | 69.35 | | 0000N | |
| (87.90) | 2 | | | | Strong flut. head { Hale 7088.00, Spot 7087.90 |
| (99.83) | 0–1 | 99.812 | | 0000N | Fluting line. Hale 7099.77 |
| (7111.70) | 0–1 | 7111.730 | A | 000N | Fluting line. Hale 7111.73 |
| (25.83) | 2 | | | | Strong flut. head { Hale 7125.88, Spot 7125.89 |
| 39.24 | 2–3 | 39.215 | | 000 | |
| 88.90 | 0–1 | 88.875 | | 0000N | Fluting line? |
| 90.22 | 2 | 90.159 | | 000 | |
| 7209.78 | 5–6 | 7209.78 | A, – | 3 | |
| 16.53 | 3 | 16.482 | A | 000 | |
| 45.15 | 4 | 45.152 | | 00 | |
| 52.06 | 4 | 52.032 | | 000N | |
| 54.12 | 0–1 | | | | |
| 66.60 | 0 | | | | May be fluting lines |
| 71.83 | 0 | | | | |
| 74.12 | 0 | | | | |
| 7300.05 | 0 | | | | |
| 18.78 | 1 | | | | |
| 45.12 | 3 | | | | These wave-lengths above 7252 were obtained by extrapolation and may be incorrect to 0.2 or 0.3 of a tenth-meter |
| 58.15 | 2 | | | | |
| 7364.50 | 2 | | | | |

for other lines. The wave-lengths as determined by the prism were confirmed within about 0.1 of a tenth-meter. In the reduction of the prismatic spectra in the extreme red the Cornu-Hartmann formula was used over quite a long range ($\lambda$ 6861.77 to 7209.78), and it is interesting to note the close agreement between the wave-lengths obtained in this region with a single prism and those obtained with the grating.

The titanium spectrum was always obtained as a mixture of the line and fluted spectrum, and possibly a few of the tabulated lines, in addition to those doubtful cases indicated, belong to the fluting structure.

The impurity lines identified on the plates were due to calcium, vanadium, barium, and lithium, and they have been eliminated as far as possible in the appended table. The wave-lengths of the lines from 5866 to 6600 have been chiefly obtained from measurements of prismatic spectra, while the wave-lengths from 6600 onward have been derived from the grating photographs.

I am indebted to Professor Fowler for valuable assistance during the course of the investigation.

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# THE MERCURY PARABOLOID AS A REFLECTING TELESCOPE

By R. W. WOOD

The idea of utilizing the principle that the free surface of a liquid, rotating with a uniform velocity, assumes the form of a paraboloid, in the construction of a reflecting telescope is not new. Mercury telescopes have been suggested from time to time for the past half-century, and in the early seventies one was constructed by Mr. R. C. Carrington, on Frensham common. So far as I know, however, no serious attempt has ever been made to devise a method of rotating the fluid without, at the same time, communicating jars to it. The scheme has always been regarded as a joke by astronomers, which is not surprising when one considers the perfection required of an optical surface and the ease with which ripples are set up on a free surface of mercury.

The idea of using a rotating magnetic field to communicate motion to the circular basin of mercury occurred to me last summer, and becoming interested in the problem, more as a mechanical puzzle than anything else, I determined to ascertain if possible whether, after all, the difficulties were insurmountable. A small instrument seven inches in diameter was constructed and arranged to be driven by a revolving ring of small magnets, which pulled a concentric ring of magnets fastened to the dish around with it. The rotor, as we may term the outer ring, was mounted on a support which was completely insulated from the mercury basin, and the jars which it received from the motor along the driving belt were in this way prevented from reaching the liquid. It was of course found impossible to learn much about the instrument in the city, but I convinced myself that the ripples on the surface were no worse when the basin was revolving than when it was at rest.

On removing to East Hampton, L. I., for the summer, where I have fitted up a small laboratory, the experiments were commenced in earnest. The mirror was mounted upon a small brick pier and sur-

rounded by an empty barrel to protect it from air currents. It worked so much better than I had expected, in spite of its poor workmanship, that I at once determined to have a larger instrument constructed of the finest workmanship possible. A week or two was spent in experimenting with the small model, in order to learn as much as possible about its peculiarities and the sources of trouble, after which drawings for a 20-inch instrument were prepared and submitted to Messrs. Warner and Swasey of Cleveland. Mr. Warner very kindly gave his personal attention to the matter, modified the drawings, introducing some new features, and had the work commenced at once. The 7-inch instrument was made from an old casting which I found in the junk heap of the laboratory. It was in the form of a circular disk surmounting a short cylindrical pillar. The disk was turned out into a shallow flat-bottomed basin, and the pillar bored out with a hole about 1 cm in diameter, conical at the end. The whole was supported on a steel cylinder, turned to a cone at the top, with a bearing surface about 5 mm wide at the bottom, the construction adopted in the "turn-tables" made for finishing microscope slides. It was found with this instrument that there were four distinct sources of ripples:

*a*) Jars from the driving mechanism, i. e., motor, speed-pulleys, etc. These were eliminated by the magnetic clutch previously described.

*b*) Jars due to the grinding-together of the bearing surfaces of the mercury basin itself. These were always present in the first instrument as the surfaces were not polished. It was hoped that better workmanship would eliminate them, as was found to be the case.

*c*) Imperfect leveling of the instrument, which sets up a sort of tidal wave; easily overcome provided the axis of rotation is accurately perpendicular to the flat bottom of the dish.

*d*) Variations in the velocity of rotation, which give rise to very troublesome waves. Of these I shall speak more at length presently. Mr. Warner suggested the use of two slightly tapering conical surfaces for the cylindrical base of the dish and the supporting pillar, with a central shaft of steel, by the elevation of which the dish could be lifted a trifle from the conical supports.

The construction will be readily understood by reference to Fig. 1. A plug of hardened steel *A* was inserted in the iron basin, which was screwed to the top of the conical pillar *K*. This plug rested upon a second plug of hardened steel *B*, which could be raised or lowered by means of the screw *C*, turned by means of the nut *D*. The bearing surfaces, indicated by the arrow *E*, were ground flat and accurately perpendicular to the axis of rotation. The weight of the basin was

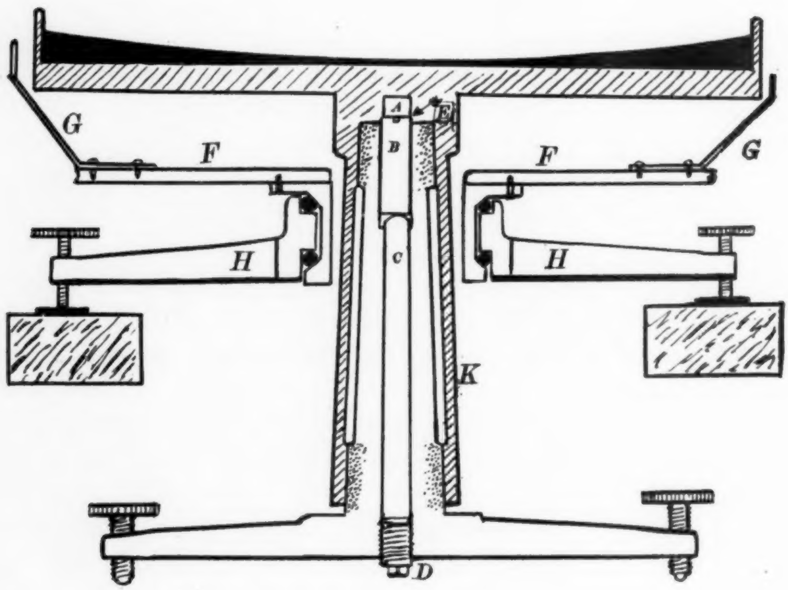


FIG. 1

carried by the steel plug, but there were in addition two conical bearings (dotted in the diagram) which kept the instrument level during rotation. By lowering the screw *C* the weight of the basin could be thrown wholly upon the conical bearings. In this position it could be turned only with some difficulty, owing to the increased friction. The best position was secured by raising the screw by an amount just sufficient to abolish this friction. The driving mechanism or rotor consisted of a large wooden pulley *F*, mounted on ball bearings, carried on a tripod *H*, which was supported independently of the

rotating basin. The original plan was to apply the power of the rotor to the dish by means of a system of magnets, but it was found that fine elastic threads of rubber answered equally well. These were fastened to the brackets *G*, six in number, and attached to the rim of the basin. They applied their force tangentially, and did not appear to transmit the jars of the rotor to the heavy iron basin. A photograph of the instrument, the driving pulleys, and the motor is shown in Fig. 2.

Preliminary experiments were made with the instrument mounted on a brick pier in the laboratory before its installation at the bottom of the cement pit. Ripples were always present here, as it was impossible to secure the complete insulation of the rotor, and the jars from the motor were transmitted to the pier through the floor. A good deal was learned, however, about the adjustments of the instrument in spite of the unfavorable conditions. Accurate leveling is of the greatest importance. The dish was first leveled by means of a spirit level, the final adjustments being made optically.

A Nernst lamp, with its filament vertical, was mounted on the ceiling of the laboratory, at a height of about 12 feet above the mercury surface. At a speed of rotation of about one turn in 3.5 seconds the image was formed at a point where it could be reached easily and examined by standing on a ladder. If a sheet of paper was held a few inches below the focal point, the circular ripples were clearly visible. The bright center of the ripple system did not, however, coincide with the center of the convergent cone of rays. By turning the leveling screws it could, however, be brought accurately to the center which gives us a first approximation to the true level desired. The ripples became much less conspicuous as soon as this operation was performed, and it was speedily discovered that they were in large part due to a sort of tidal wave resulting from imperfect leveling. If the basin is not level the depth of the mercury is slightly greater on the lower side of the basin. This point of maximum depth is carried around to the high side by the rotation of the dish, and the mercury seeks to establish its level again. If the eye is placed in the focus a curious sort of spiral eddy is seen, which is caused by this continual readjustment of level. With the instrument in approximate adjustment, the surface appears perfectly smooth and free from ripples except under especially favorable

illumination. The appearance of the extra-focal image as the level is improved is shown in Fig. 3 (upper row). These pictures were made by inclosing a plate in a holder provided with a shutter giving an exposure of about $\frac{1}{25}$ second. In the lower row we have images obtained under the same conditions with the plate almost at the focal point. A star seen under these conditions shows a very marked coma, which can be abolished by turning the leveling screws a proper amount. When the mirror is accurately level, if the image is received on a piece of paper, and one of the leveling screws turned a trifle, the image moves slightly to one side, just as it would do if we tilted slightly a solid mirror. The image develops a coma, of course, but it is interesting that an actual displacement of the point of light can be obtained in this way, a circumstance which one would hardly have anticipated.

Some experiments have been made with two concentric dishes, the inner floating on the liquid contained in the outer. This expedient does not appear to lessen the effect of jars coming from the ground though it improves matters in some other respects. In the end it may be found necessary, though there are certain difficulties attending its use.

One thing which struck me as especially interesting was the slowness with which the velocity of the dish was communicated to the fluid. The mercury picks up its velocity first at the rim, a circular zone of constant velocity and focus crawling in gradually from the edge toward the center. If the eye is placed above the center of the dish a little above a focus, one sees a narrow ring of light of dazzling intensity converge gradually from the rim toward the center. It required over two minutes for all parts of the fluid mass to acquire a constant angular velocity.

The appearance of the room as viewed in the mirror while it is getting up speed is very striking. The rafters of the roof recede to an enormous height and the whole room appears to expand in a most remarkable manner. The appearance of the mercury surface in rotation, and its freedom from ripples are shown in Fig. 4. The distortion of the reflected image is of course due to the astigmatism which results from oblique incidence.

I have had the mirror operating with a focal length of less than three feet, at a velocity of one turn in three seconds, i. e., with a ratio

of focus to aperture of 1.7. The dish was not deep enough to make a higher velocity possible, but I have had smaller mirrors running with a focal length of less than one-fourth of the aperture. These very deep paraboloids might prove useful where a great concentration of energy was desirable, as in bolometric determinations of the heat of the stars or the sun.

After completing the preliminary experiments in the laboratory preparations were made for mounting the instrument below ground. An old well-house adjoining the barn was transformed into an observatory, a new floor being laid and a large scuttle cut in the roof. The brick well, which was no longer used, was filled up with large granite blocks and Portland cement, to a height of about two feet above the water level, after which it was lined with cement, forming a water-tight pit about 14 feet deep and 30 inches in diameter. The granite and cement bottom formed a very solid foundation for the mirror to stand upon. A second pit was sunk about six feet from the well, just outside of the well-house, and the two put in communication by means of a tunnel at the bottom. It was the original plan to drive the mirror by means of a pulley-belt running down the side of the cement pit, but as it was found that it was necessary to make very precise adjustments while the mirror was running, the second shaft became necessary. The mirror was mounted upon three blocks of cast iron which rested on the cement floor, and the driving mechanism or rotor was carried upon a wooden *Y*-beam which passed under the floor of the tunnel and was securely anchored to brick piers in the second pit. The electric motor, driven by a 110-volt alternating current, and the speed-pulleys were mounted in the second pit, the belt passing under the floor just above the *Y*-beam and around the wooden pulley of the rotor. The tunnel was lighted by an electric lamp.

The general arrangement of the apparatus is shown in Fig. 5. It will be seen from the diagram that the slight jars from the rotor can reach the revolving mirror only by passing back to the second pit along the *Y*-beam, thence through the brick piers and back through seven or eight feet of sand to the massive granite foundation. It is needless to say that they were quite imperceptible at the end of their journey. Within the well-house, and close to the mouth of the pit

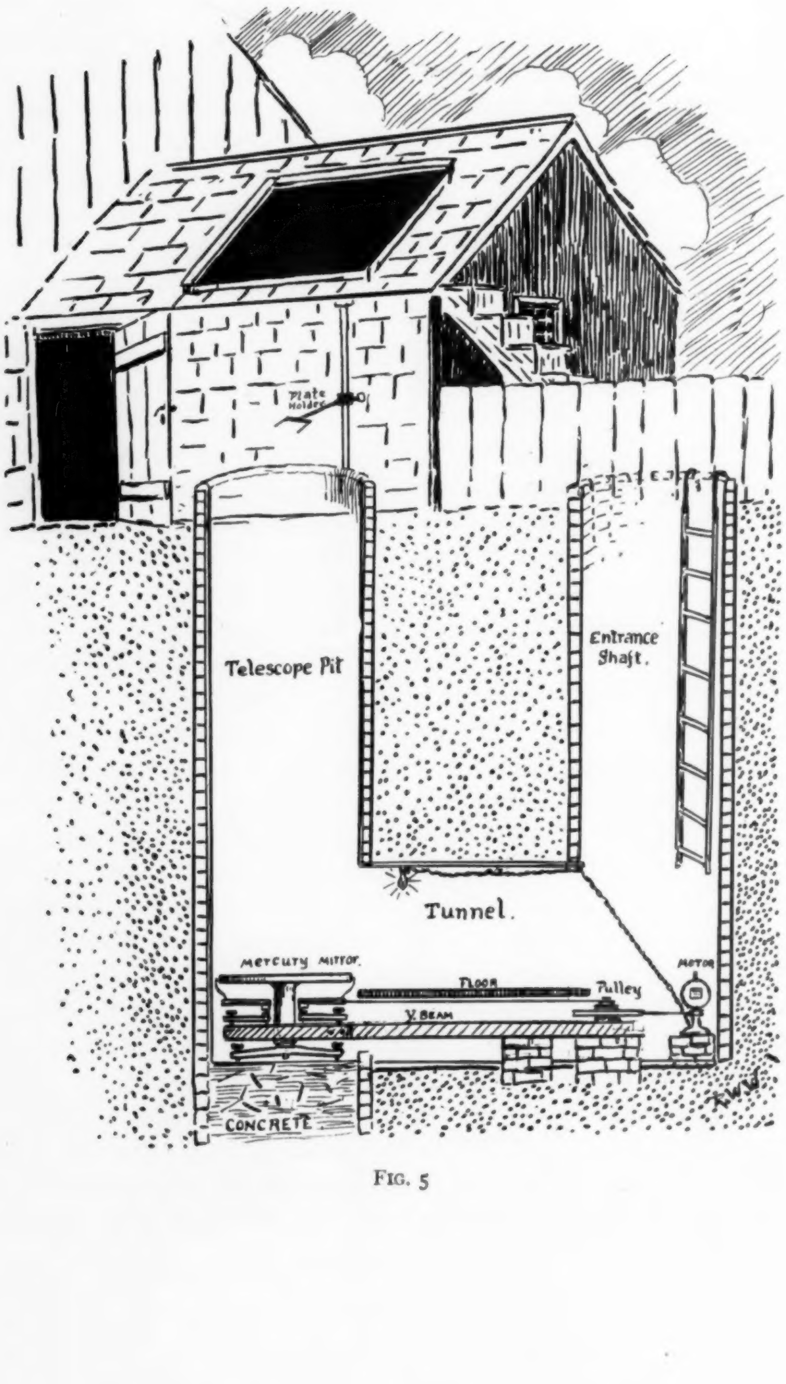
Plate Holder
Telescope Pit
Entrance Shaft.
Tunnel.
Mercury mirror.
Floor
Pulley
Motor
Y. Beam
Concrete
RWW

FIG. 5

a vertical wooden cylinder was mounted with a horizontal arm carrying a plate-holder. A photographic plate could, by means of this simple device, be held fixed in any position above the pit, for making star trails. Visual observations were made by standing at the mouth of the pit, or upon the roof of the house, according to the focal length used, and holding an eyepiece in the hand.

It was first necessary to determine to what extent vibrations, other than those caused by the motion of the mirror, affected the mercury surface. A five-inch refractor, with an excellent objective figured by Clark, was mounted at the side of the pit, and directed downward at the mercury surface. The star images were perfectly sharp, even when the motor was driving the rotor, showing that no trouble was to be anticipated from tremors running back along the *Y*-beam. The approach of a horse and carriage could be detected, however, when it was an eighth of a mile away, and the footsteps of a person running across the lawn fifty yards from the telescope house caused a perceptible vibration of the image. Occasional disturbances were noticed due to live stock in the barn, and after every storm, vibrations were found resulting from the pounding of the surf on the beach a quarter of a mile distant.

At the time when the experiments were commenced the Milky Way was in the zenith at nine o'clock, and its appearance when I observed it from the roof of the house the first time the motor was started amply repaid me for all my trouble. No eyepiece was used, the star images appearing in space about three feet above the mouth of the pit. They appeared perfectly sharp and quite steady, since the cone of light entering the pupil of the eye under these conditions comes from but a small part of the mirror's surface. On examining the images with the eyepiece it was at once seen that the focus changed periodically through a range of perhaps an inch and a half. On viewing the images in space with both eyes they were seen to be rising and falling rhythmically, dancing up and down like will-o'-the-wisps. There were moments when they were quite sharp in the eyepiece, but even then they were not quite stationary, moving about in a sort of Lissajou figure, perhaps a millimeter in diameter. The nature of the disturbance on the mercury surface could be detected by placing the pupil of the eye in coincidence with the star image. The

whole surface of the mirror was then seen filled with a blaze of light and the ripples were plainly visible. It was at once apparent that they were not "jar" ripples, for these always appear as narrow concentric circles surrounding the center of the mirror. They often appear nearly stationary owing to interference. The ripples which were causing the trouble appeared to start at definite points on the rim and spread out across the surface, i. e., their centers were on the rim and not at center of the basin. After a good deal of experimenting it was finally found that these ripples were due to slight periodic variations in the velocity, which caused a "slip" between the mercury and the iron rim of the basin. At points where there was any roughness, or where grains of sand happened to lie, the ripples started out.

The periodic change of focus was also found to be due to the variation of the velocity, which occurred once in every revolution of the dish. This change in the velocity is very small, and it is not communicated to the fluid, for, as we have seen, two minutes are required for the establishment of the steady state, starting from rest. When the reduction in speed occurs, though it lasts but a fraction of a second, the liquid at the rim falls a trifle, rising at the center. The propagation of the change of curvature from the rim to the center is almost instantaneous, and not at all like the slow crawling-in of the zone of constant curvature seen when starting the dish. This trouble can probably be overcome to a great extent, if not completely, by making one or two changes in the construction of the instrument. Its presence was detected by fastening a small square of paper to the rim of the basin, and another immediately opposite it on one of the supports to which the rubber bands were fastened. If the dish turned at the same velocity as the rotor, the two pieces of paper should remain in juxtaposition: it was found, however, that once in every revolution, the piece of paper on the basin first lagged behind, and then gained upon, the one fastened to the rotor. A little further experimenting showed that this variation was due to the fact that the friction was a little greater at this point, that is, the force necessary to start the basin varied with the position of the dish. When the dish, in its revolution, reached the position of maximum friction, it lagged a little behind the rotor, the rubber threads stretching a trifle. As soon

as the point was passed and the friction diminished, the increased tension on the threads caused a slight acceleration, the basin catching up with, and even running a little ahead of, the rotor. The actual slip between the rotor and the basin only amounted to about a centimeter, but this was quite sufficient to account for all of the disturbances, as was shown by intentionally varying the velocity by small amounts. This trouble was not found with the small model first constructed, which spun on a point instead of a flat surface. The friction is much greater in the latter case, owing to the increased length of the "lever-arm" of the moving surface elements which rub together. I am at the present time taking graphs of the periodic variations in the velocity by means of a pen carried by the rotor, and a small improvised chronograph carried by the basin. These furnish a good deal of information about the effects of raising or lowering the dish, slackening the driving belts, etc. Much better results have been obtained by introducing a small steel ball between the two bearing surfaces. I am of the opinion that a conical point is the best form of support, and as soon as the chronograph experiments are finished, alterations in the construction will be commenced.

A number of star trails were made with the mirror running with a 15-foot focus. One of these ($\gamma$ *Cygni*) is reproduced in Fig. 3 (lower figure). We obtain a dotted line of bright points, owing to the periodic change in the focus. The points are fairly small, in fact not so very much larger than their theoretical size, and when we consider that they were formed by light-rays concentrated by a rotating mercury surface 20 inches in diameter, at a distance of 15 feet from the photographic plate, they appear to me to be surprisingly good. I feel very confident that the velocity variations can be abolished. It may be necessary in the end to use a motor, the speed of which is controlled by a clock, the plan so successfully worked out by Mr. Gerrish of the Harvard Observatory.

I had no trouble in separating double stars $5''$ or $6''$ apart, and once I felt pretty sure that I had momentary glimpses of the resolution of both $\epsilon^1$ and $\epsilon^2$ *Lyrae.*

The great nebula of *Andromeda* and the large cluster in *Hercules* both cross the zenith at East Hampton, and will furnish interesting objects next summer when the experiments are taken up again. I

have already had some splendid views of the nebula, which appeared almost as satisfactory as in a large refracting telescope.

I have not as yet convinced myself that the mercury telescope will be of use in astronomical observations, even if it is possible to bring it to a high degree of perfection. Its great disadvantage lies in the fact that its observations are confined to a narrow region surrounding the zenith. On the other hand, it has the great advantage of a focus which may be made to vary over as wide a range as we please, and its cost is very small. The 20-inch instrument, with its motor, cement pit, entrance shaft, tunnel, etc., cost only $200, not including the mercury. In the case of small instruments, not over three feet in diameter, it may be feasible to use an auxiliary plane mirror driven by clock work or by hand, for reflecting the light from any part of the sky to the bottom of the pit. An instrument of large size could in this way be placed within the means of any amateur observer, and valuable data might be secured. Whether a gigantic reflector mounted in a suitable latitude would enable us to see more of planetary detail, I do not feel prepared to say. We could in this way combine great focal length and immense light-gathering power, but we should still have the atmospheric disturbances to contend with.

As I stated at the beginning of the paper, the work was undertaken purely as a diversion for the summer months, and if further experimenting shows us that a perfect reflector can eventually be obtained in this way, it will then be time enough to consider whether it is worth while to attempt the construction of a much larger instrument. Then, too, we may be able to discover some substance which can be fused, rotated, and allowed to solidify while revolving with a constant velocity. Refiguring of the surface would very probably be found necessary.

### DAMPING OF THE RIPPLES

In the case of a partially viscous substance, there would be much less trouble from ripples. Last summer I made a fairly good mirror of gelatine, and I am at present looking about for a more permanent medium. I shall be very glad to receive any suggestions about easily fusible media which will solidify with an optical surface. The cost of the experiments was defrayed by a grant of $200 from the Elizabeth Thomson fund.

While it has seemed best to bring the instrument to the highest degree of perfection before attempting to devise methods of destroying any residual ripples, some experiments have already been made in this direction. Two methods suggest themselves. We may use a copper vessel, turned to an approximate paraboloid, and use a thin film of mercury (as in the case of the artificial horizon). The rotation would then have the effect of figuring the surface, the thinness of the mercury layer preventing the propagation of ripples. This method has not yet been tried, as the objections to it are obvious.

The ripples can be damped to an appreciable degree by covering the mercury surface with a thin layer of water, which assumes the same curvature as the mercury. Glycerin damps them completely if they are not of too great amplitude to start with. Even if they are of large magnitude, the effect of the glycerin is surprising. With my seven-inch mirror, mounted on a table which was jarred to such an extent by the machinery in the room below that the vibrations could be felt with the hand, I have obtained almost as good definition as at East Hampton with every precaution taken to avoid jars. The action of the glycerin is well illustrated by Figs. 6 and 7, which are photographs of the filament of an incandescent lamp taken with the mirror. The mirror was running at such a speed that the image was formed about a meter above the surface. The belt from the motor drove the dish directly, no rotor being used, and the jars from the city traffic and machinery were so great that the ripples could be seen even in the most unfavorable illumination. Fig. 6 was taken without, Fig. 7 with, the glycerin cover.

Two small dishes of mercury were then placed on the floor, which was violently shaken by shafting attached to the floor beams in the shop below. The jarring was so great as to be unpleasant. The mercury in one dish was covered with glycerin to a depth of about 4 mm. The light from an arc lamp was then reflected from the two dishes directly to a photographic plate, exposed behind a "focal-plane" shutter. The mercury waves can scarcely be seen in the image (Fig. 8) obtained from the glycerin-covered surface, while the other image (Fig. 9) speaks for itself.

It seems quite possible that the glycerin cover may prove useful in making artificial horizons. Inasmuch as my twenty-inch mirror

now runs with so little error and with such insignificant ripples that the maximum deviations from perfect focus amount only to a millimeter or less (and this with a 5 meter focal length), I feel very confident that the glycerin cover will make the image practically perfect. So far as I can see now, the glycerin cover will introduce no disturbances, provided its temperature can be kept constant. Even with a variable temperature, the surfaces of equal refractive index would be parallel to the paraboloid, and would probably not give trouble. Stirring the fluid would of course produce striae, if its temperature varied, but stirring only occurs when starting and stopping the mirror. Things would probably come to a steady state in ten or fifteen minutes. I am now making experiments to see whether solid mirrors can be produced by the centrifugal method. Various methods have occurred to me. We may be able to take a cast of the mercury surface by pouring a hot viscous liquid over it, which will solidify on cooling. I have already obtained in this manner pretty good convex paraboloids of resin. These casts could be electrotyped and silvered. If a suitable material can be found, and the electrotypes do not warp, I see no reason why large mirrors cannot be made in this way at very small expense. At all events the method seems well worth trying.

Johns Hopkins University
December 1908

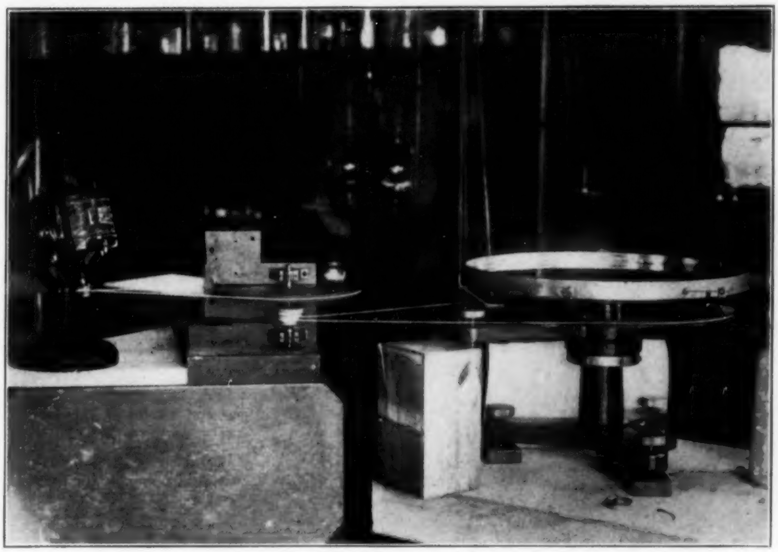

FIG. 2

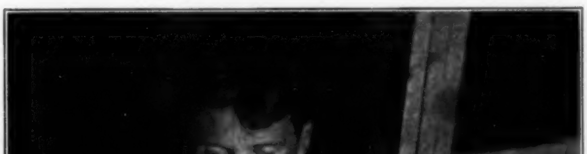

FIG. 4

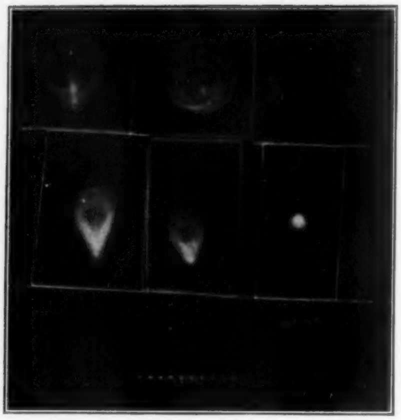

FIG. 3

FIG. 6

FIG. 7

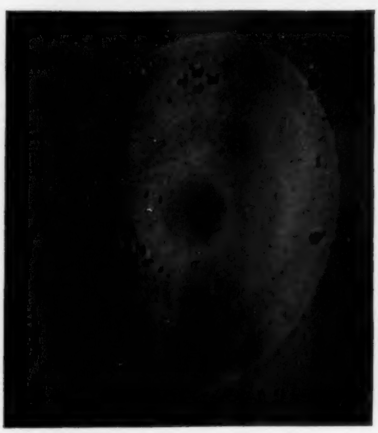

FIG. 8

FIG. 9